Forecasting Market Drawdowns in Q1 2025: A Volatility Analysis of Major U.S. Indices Using GARCH, EGARCH, VaR, EVT and Seasonal Decomposition Models

Abstract

In this study have been researched the volatility patterns of major stock market indices such as (DJIA, S&P500, NDX, and Russell 3000) in discrete time-series 2007/2024, with a focus on analysing the Q4/2024 volatility patterns in order to gather data points able to form empirical bases for a potential Q1 2025 stock market drawdown, according also to seasonality effects. The working paper provides insights into VaR violations and volatility patterns clustering between Q3-Q4 2024 providing then a solid base and data points to achieve an expected Q1 2025 drawdown forecast. These econometrics and Volatility pattern methods can be utilised in forecasting unforeseen stock market volatility and the feasibility of extreme fat tail losses withstanding seasonality in stock market scenarios.

Introduction

Volatility and volatility patterns, as measures of the dispersion of asset returns, are critical concepts in financial markets. Volatility serves as an important indicator of market risk and has a central role in portfolio management, option pricing, and risk assessment. Accurate and precise modelling of volatility is essential for forecasting stock market corrections, which can have profound implications for investors and policymakers. Traditional models, such as GARCH and ARIMA, have been widely used to capture time-varying volatility, while advanced techniques like Extreme Value Theory (EVT) provide insights into tail risk during extreme market conditions. However, modelling volatility remains challenging, particularly during periods of financial stress, such as the 2008 global financial crisis and the 2020 COVID-19 market crash. This study aims to address these challenges by leveraging a comprehensive set of volatility models to identify patterns in Q4 2024 and forecast potential market drawdowns in Q1 2025.

The primary objective of this study is to identify volatility patterns in major market indices—specifically the Dow Jones Industrial Average (DJIA), S&P 500 (GSPC), NASDAQ 100 (NDX), and Russell 3000 (RUA)—during Q4 2024 and use these patterns to forecast potential market drawdowns in Q1 2025. To achieve this, we employ a combination of advanced volatility models, including GARCH, EGARCH, ARIMA, SARIMA, and Extreme Value Theory (EVT). By integrating these models, we aim to provide a robust framework for understanding short-term and long-term volatility dynamics, as well as tail risk during extreme market conditions.

This research is highly relevant for a wide range of stakeholders in financial markets. For investors and portfolio managers, accurate volatility forecasts can inform risk management

strategies, asset allocation decisions, and hedging practices. Policymakers can use these insights to monitor market stability and implement measures to mitigate systemic risks. Furthermore, the timeliness of this study—focusing on Q4 2024 and Q1 2025—makes it particularly valuable for anticipating and preparing for potential market downturns in the near future. By combining multiple volatility models and incorporating tail risk analysis, this study provides a comprehensive approach to understanding and forecasting market behaviour.
The remainder of this paper is organized as follows. Section 2 provides a review of the relevant literature on volatility modelling, market drawdowns, and the application of advanced techniques such as GARCH, ARIMA, and EVT. Section 3 describes the datasets and methodologies used in this study, including the calculation of annualized volatility, the fitting of GARCH and ARIMA models, and the application of Extreme Value Theory (EVT) for tail risk analysis. Section 4 presents the empirical results, including volatility patterns in Q4 2024 and forecasts for Q1 2025. Section 5 discusses the implications of these findings for risk management and market stability. Finally, Section 6 concludes the paper with a summary of key findings and suggestions for future research."

Literature
Volatility Models (GARCH, EGARCH, ARIMA, SARIMA)

- ***Engle, R. F. (1982)***
- ***Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation***
- ***Bollerslev, T. (1986)***
- ***Generalized Autoregressive Conditional Heteroskedasticity***
- ***Box, G. E. P., & Jenkins, G. M. (1976)***
- ***Time Series Analysis: Forecasting and Control***

Seasonal Volatility Decomposition

- **Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2003)**
  Modeling and Forecasting Realized Volatility
- ***Ghysels, E., Santa-Clara, P., & Valkanov, R. (2006)***
  Predicting Volatility: Getting the Most Out of Return Data Sampled at Different Frequencies

Value-at-Risk (VaR) and Its Applications

- ***McNeil, A. J., Frey, R., & Embrechts, P. (2015)***
- *Quantitative Risk Management: Concepts, Techniques, and Tools*
- ***Jorion, P. (2006)***

- *Value at Risk: The New Benchmark for Managing Financial Risk*

Extreme Value Theory (EVT) and Generalized Pareto Distribution

- ***McNeil, A. J. (1997)***
- *Estimating the Tails of Loss Severity Distributions Using Extreme Value Theory*
- ***Coles, S. (2001)***
- *An Introduction to Statistical Modeling of Extreme Values*

Market Drawdowns and Volatility Forecasting

- **Danielsson, J., & de Vries, C. G. (2000)**
  Value-at-Risk and Extreme Returns
- ***Hansen, P. R., & Lunde, A. (2005)***
  A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)
- ***McNeil, A. J., & Frey, R. (2000)***
  Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: An Extreme Value Approach
- ***Bali, T. G., & Weinbaum, D. (2007)***
  A Comparative Study of Alternative Extreme Value Volatility Estimators

Literature Review

The ARCH and GARCH models, introduced by Engle (1982) and Bollerslev (1986), marked a significant advancement in volatility modelling by capturing time-varying volatility and clustering effects. However, these models assume symmetric responses to shocks, which led to the development of EGARCH by Nelson (1991) to account for leverage effects. While GARCH-family models excel in modelling volatility, ARIMA and SARIMA models are better suited for capturing linear dependencies and seasonal patterns in returns. Despite their strengths, these models often struggle to capture extreme events and tail risk, which are critical for forecasting market drawdowns. This highlights the need for integrating multiple models to address these limitations.

Seasonal volatility decomposition plays a crucial role in identifying periodic patterns in financial markets, which can significantly impact volatility forecasting. Andersen et al. (2003) demonstrated the effectiveness of seasonal decomposition in high-frequency financial data, showing that ignoring seasonal effects can lead to biased volatility estimates. However, traditional decomposition methods often struggle to capture complex seasonal patterns, particularly in the presence of structural breaks or extreme events. This underscores the need for integrating seasonal decomposition with advanced volatility models to improve forecasting accuracy.

Value-at-Risk (VaR) has become a cornerstone of modern risk management, providing a quantifiable measure of potential losses over a specified time horizon at a given confidence level. Since its introduction by J.P. Morgan in the 1990s, VaR methodologies have evolved to include historical simulation, variance-covariance, and Monte Carlo simulation. However, VaR has been criticized for its inability to capture tail risk beyond the confidence level and its reliance on historical data, which may not adequately reflect future extreme events. This has led to the adoption of Expected Shortfall (ES) as a complementary measure. Despite these advancements, accurately estimating tail risk remains challenging, highlighting the need for more robust methods such as Extreme Value Theory (EVT).

Extreme Value Theory (EVT) provides a powerful framework for modelling extreme events, such as market crashes and large losses, which lie outside the range of normal observations. The Peaks-Over-Threshold (POT) method, a key approach in EVT, focuses on excesses over a predefined threshold and models them using the Generalized Pareto Distribution (GPD). McNeil (1997) and Embrechts et al. (1997) demonstrated the effectiveness of EVT and GPD in capturing tail risk in financial markets, providing a more accurate estimation of extreme losses compared to traditional methods. However, applying EVT presents challenges, such as selecting an appropriate threshold and ensuring stationarity in the data. Despite these challenges, EVT offers significant advantages in modelling extreme events and improving tail risk estimation.

Market drawdowns, defined as significant declines in asset prices, are a key concern for investors and policymakers. Previous studies, such as those by Mandelbrot (1963) and Fama (1965), have explored the relationship between volatility and drawdowns, highlighting the role of volatility clustering in predicting drawdowns. More recent studies have used advanced models like GARCH and EVT to improve forecasting accuracy. However, many of these studies focus on single models or short-time horizons, limiting their applicability to real-world scenarios. This underscores the need for a comprehensive approach that integrates multiple models and analyzes longer time horizons to improve the forecasting of market drawdowns.

Despite significant advancements in volatility modelling, several gaps remain in the literature. First, there is a lack of integration between different models, such as GARCH, ARIMA, and EVT, which limits their combined predictive power. Second, the application of seasonal decomposition in volatility forecasting has been limited, particularly in the context of extreme events. Third, existing tail risk estimation methods often fail to capture the full extent of market stress, highlighting the need for more robust approaches. Finally, many studies focus on short time horizons, limiting their ability to forecast drawdowns over longer periods. This study addresses these gaps by integrating multiple models, incorporating seasonal decomposition, and applying EVT to improve tail risk estimation, providing a comprehensive framework for forecasting market drawdowns.

Methodology

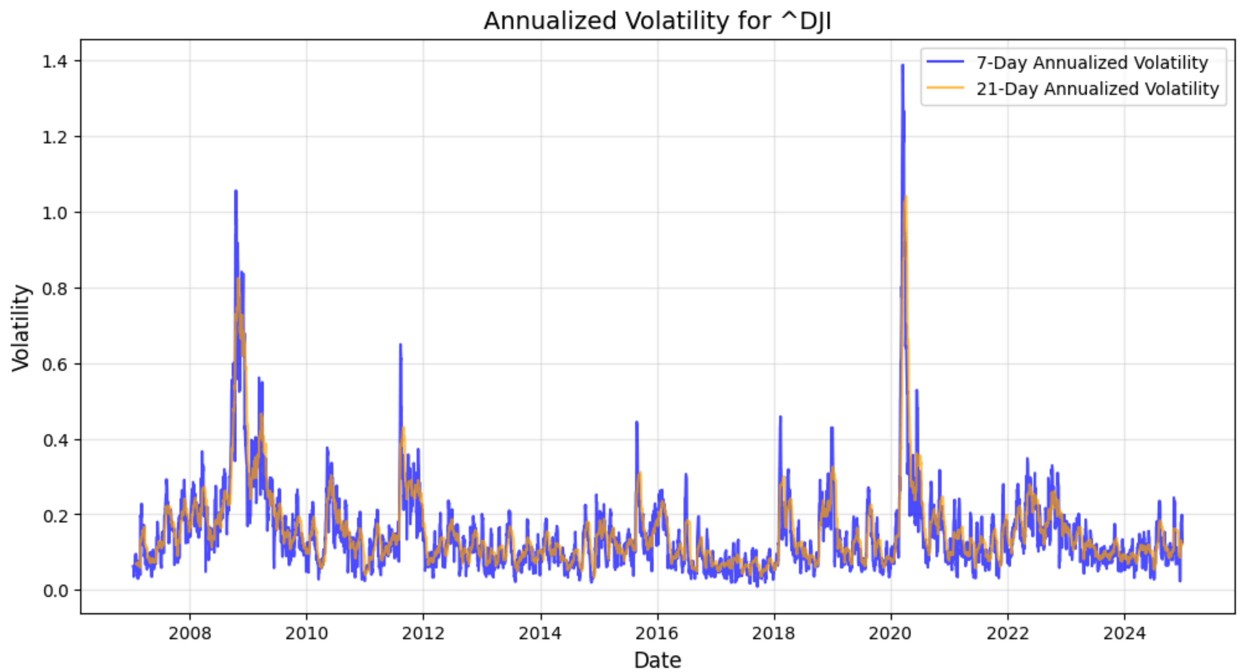
The methodology employed in this study is based on a multi-model approach, combining econometrics techniques in order to gather as many data points as possible to bring forward a Q1 2025 forecast. The starting point has been the calculation of annualised standard deviation using rolling windows, which provides a dynamic measure of short-term and medium-term volatility. ***GARCH*** *and* ***EGARCH*** *models have been utilised to estimate time-varying volatility, charting volatility clustering and leverage effects, while also utilising a narrowing timeframe approach.* ***Value-at-Risk (VaR)*** *data and parameters have been calculated at specific levels derived on the basis of the discrete time-series, while also having the total* ***VaR*** *days violations across time categorised according to* ***VaR*** *parameters. In order to gather more precise accuracy the time window has been narrowed down to Q3-Q4 2024 while interpolating VaR violations with* ***GARCH EGARCH*** *volatility and annualised standard deviation for each index. Additionally,* ***E.V.T.****, extreme value theory and* ***Generalised Pareto Distribution*** *have been utilised to focus on the tail behaviour of asset returns. Indeed, using the* ***Peaks-Over-Threshold (POT)*** *method identifies excess losses over a predefined threshold. A* ***Generalized Pareto Distribution (GPD)*** *has been fitted to the excess losses, allowing for the estimation of tail risk measures such as* ***VaR*** *and* ***Expected Shortfall(ES),*** *providing a robust framework for assessing the likelihood and magnitude of extreme market drawdowns. Seasonal volatility decomposition has been applied to identify recurring patterns and VaR days clustering. ARIMA and SARIMA models are used to capture linear dependencies and seasonal patterns in returns, with VIX and VXN included as exogenous variables to account for market expectations and future volatility. A rolling window approach is employed to ensure that forecasts are adaptive to changing market conditions. The results from all models are integrated to provide a comprehensive forecast for Q1 2025, combining volatility and VaR estimates, tail risk measures, seasonal patterns, and market expectations. Charts and data are used throughout the analysis to illustrate techniques, and ideas, present findings, and support interpretations, ensuring a robust transparent methodology.*

Starting Point: Annualised Standard Deviation

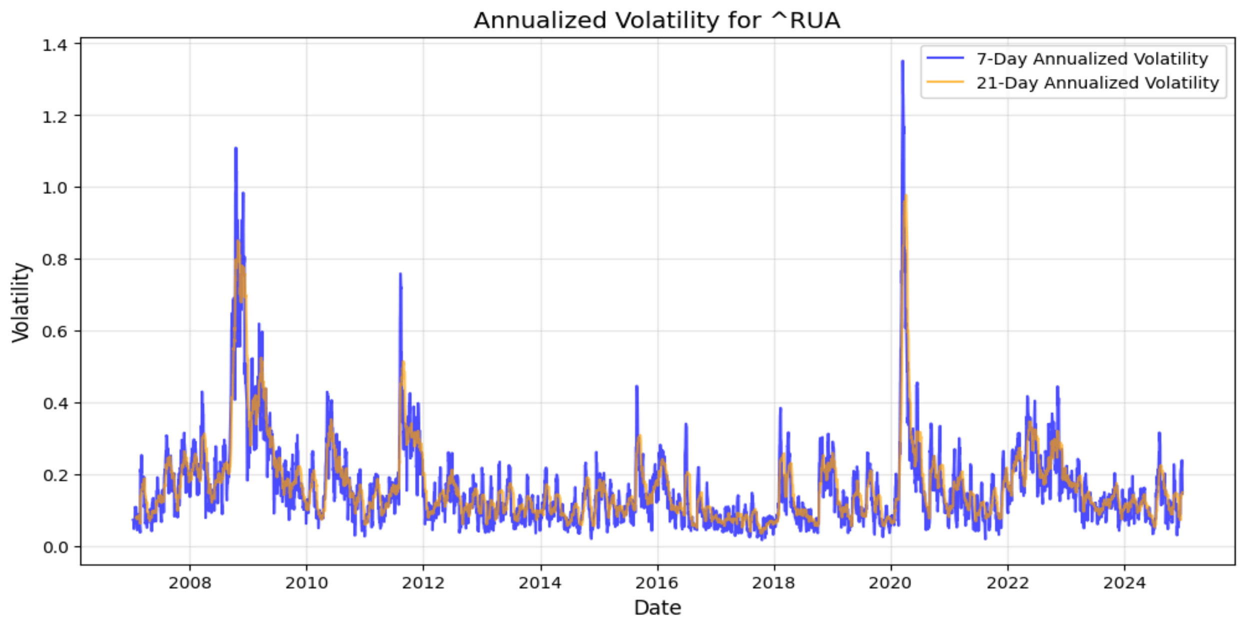
Calculate the ***annualised standard deviation*** *of daily returns for each index (DJIA, S&P 500, NASDAQ 100, Russell 3000) using rolling windows (e.g., 7-day and 21-day). This provides a dynamic measure of short-term and medium-term volatility, capturing changes over time.*

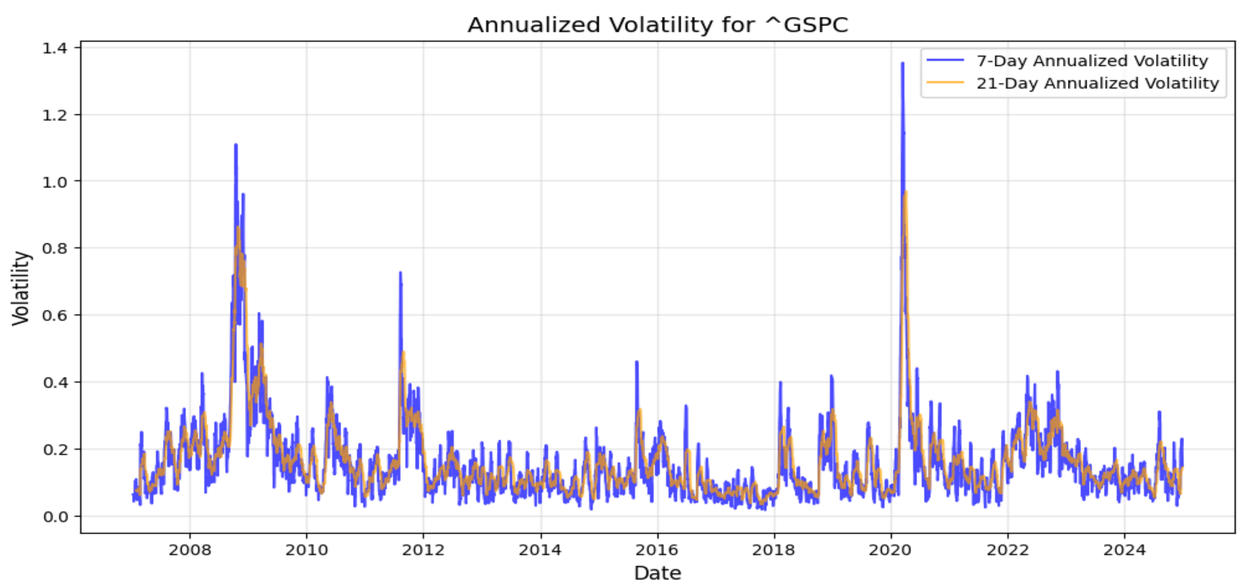
	7-Day Annualized Volatility	21-Day Annualized Volatility
Dow Jones	0.148616	0.154492
S&P 500	0.159133	0.164762
NASDAQ 100	0.191279	0.197681
Russell 3000	0.162627	0.168260

The Dow Jones, composed of more stable, blue-chip companies, shows the lowest volatility, with values of 0.149 (7-day) and 0.154 (21-day), indicating its relative resilience during periods of market stress.

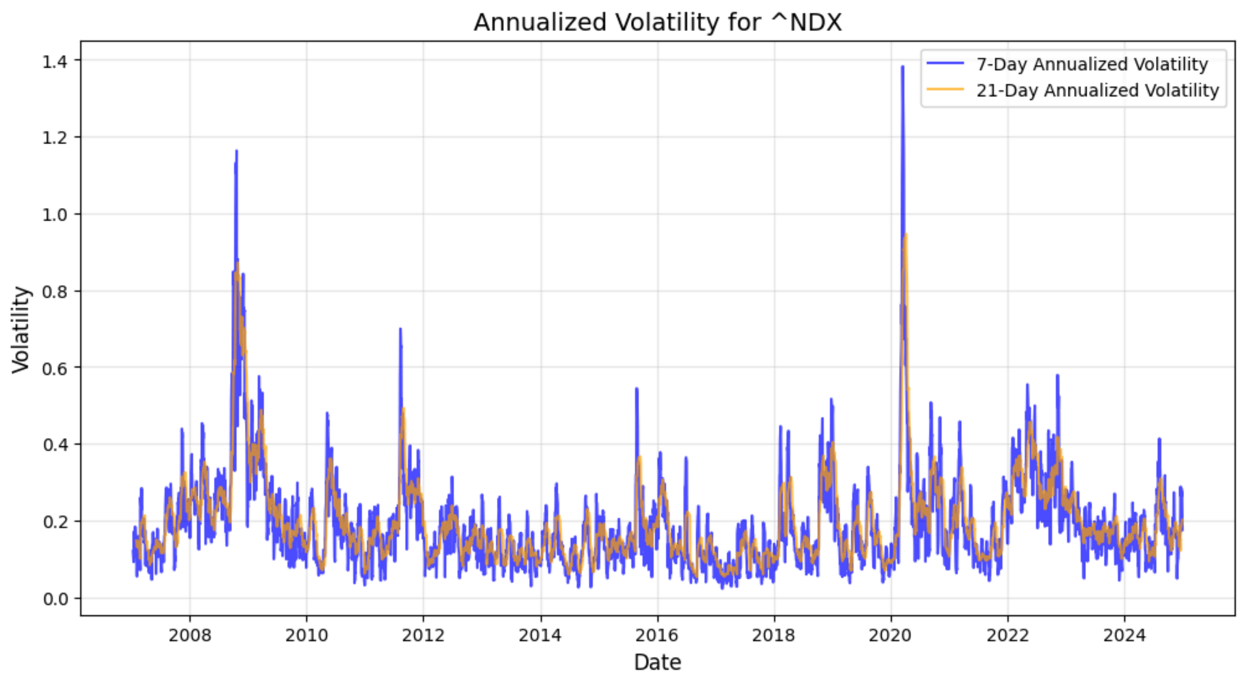


The S&P 500 and Russell 3000, representing broader market indices, display intermediate volatility levels, with 7-day and 21-day values of 0.159 and 0.165 for the S&P 500, and 0.163 and 0.168 for the Russell 3000. This aligns with their diversified compositions, which balance stability and sensitivity to market changes.





The NASDAQ 100 exhibits the highest volatility, with 7-day and 21-day annualized volatilities of 0.191 and 0.198, respectively. This reflects the greater sensitivity of technology-heavy indices to market movements and economic conditions.



Value-at-Risk data

VaR Violations Summary:

	95.0	97.5	99.0	99.9
^DJI	278	201	136	66
^GSPC	283	210	144	70
^NDX	306	215	136	60
^RUA	282	212	143	69

The Value-at-Risk (VaR) violations data provides insights into the frequency of extreme losses across the four indices: Dow Jones(DJI), S&P 500(GSPC), NASDAQ 100 (NDX), and Russell 3000 (RUA). The results summary shows that the Nasdaq 100 has the highest occurrence of VaR violations at 95% C.I. (306 violations), reflecting its higher volatility and sensitivity to market shocks. The S&P500 and Russell 3000 display intermediate levels of VaR violations, with 283 and 282 violations at 95% C.I., respectively, aligning with their broader market representation.

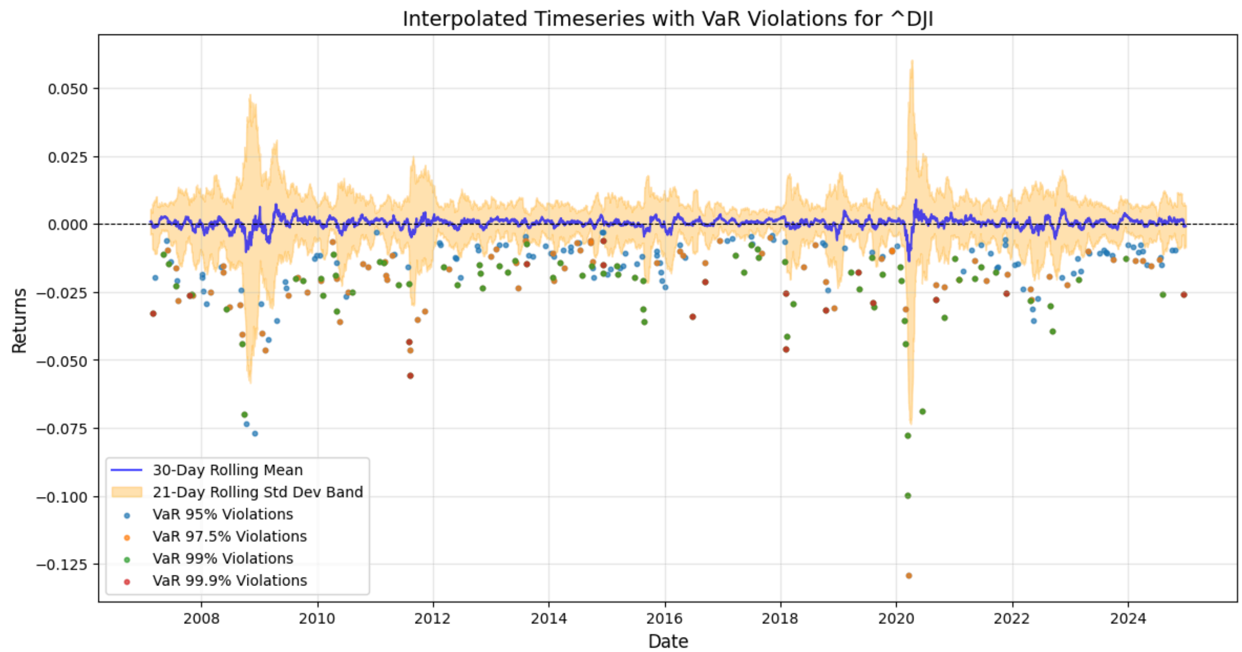
Average One-Day Drawdowns

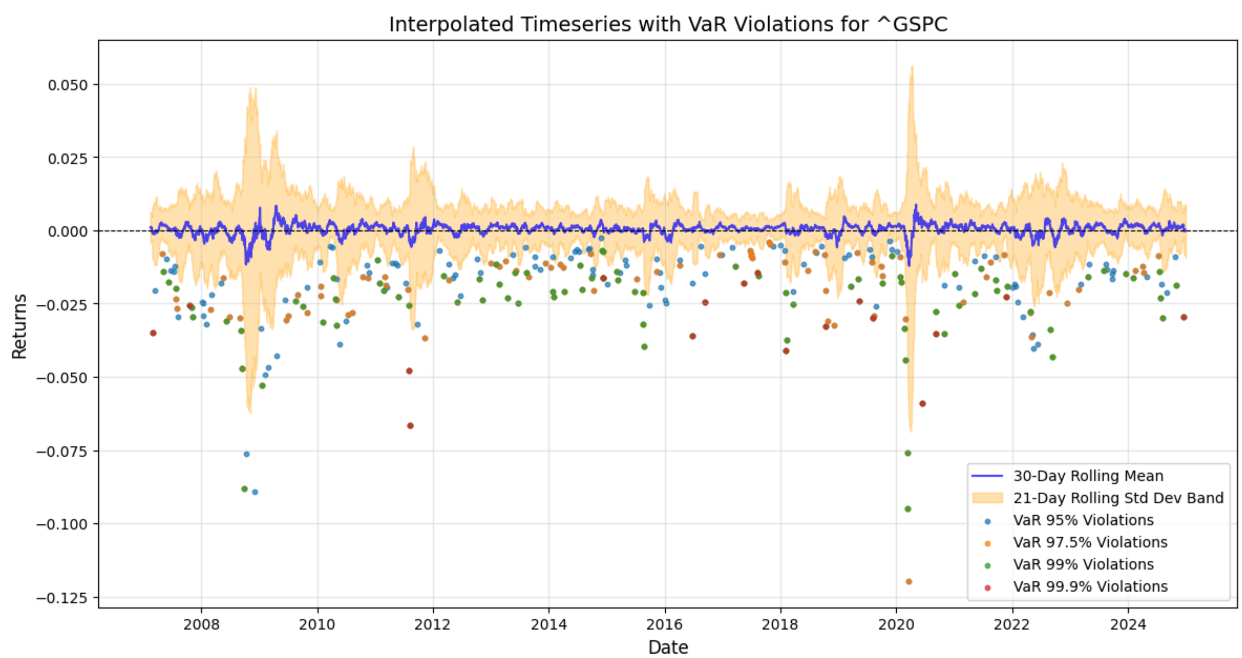
Ticker	VaR 95%	VaR 97.5%	VaR 99%	VaR 99.9%
^DJI	0.0208	0.0233	0.0258	0.0285
^GSPC	0.0223	0.0242	0.0271	0.0285
^NDX	0.0250	0.0278	0.0318	0.0377
^RUA	0.0227	0.0248	0.0271	0.0304

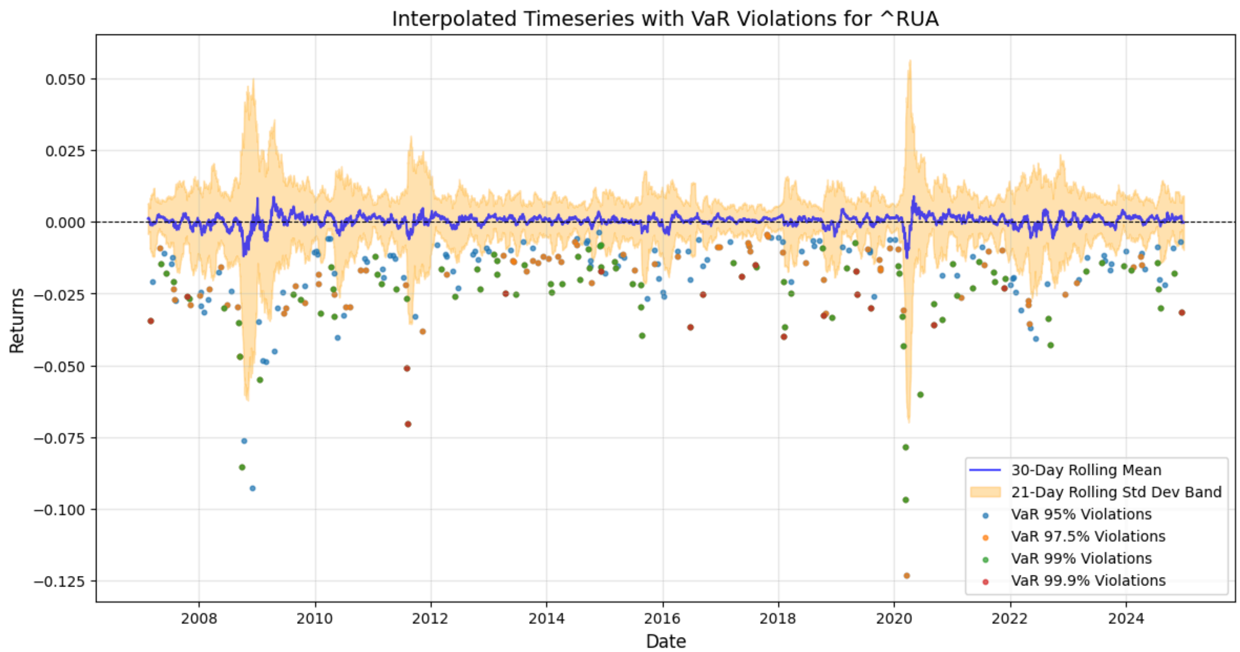
The average one-day drawdowns across four indices-Dow Jones Industrial Average (DJI), S&P500 (GSPC), NASDAQ 100 (NDX), and Russell 3000 (RUA), provide a measure of potential daily losses at various confidence levels, that have been utilised with EVT and GPD estimates of VaR and ES in fat tail scenarios. The results reveal that the NASDAQ 100 has the highest average drawdowns across all confidence levels, with a 99.9% VaR drawdowns of 0.0377, reflecting its highest volatility and sensitivity to market shocks. In contrast, the Dow Jones exhibits the lowest average drawdowns, with a 99.9% VaR drawdowns of 0.0285, indicating its relative idiosyncratic factor as a price-weighted index, exhibits relative less fat tail losses compared to market-cap-weighted indices. Heavy market-cap-weighted index as the NASDAQ100 exhibit larger VaR levels, while also the sector composition becomes an important feature as the NASDAQ100 being heavily skewed in the technology sector does impinge on idiosyncratic higher VaR levels. On a broader level, considering the S&P500 (GSPC) and the Russell 3000 (RUA) being more diversified these can provide relatively contained VaR levels.

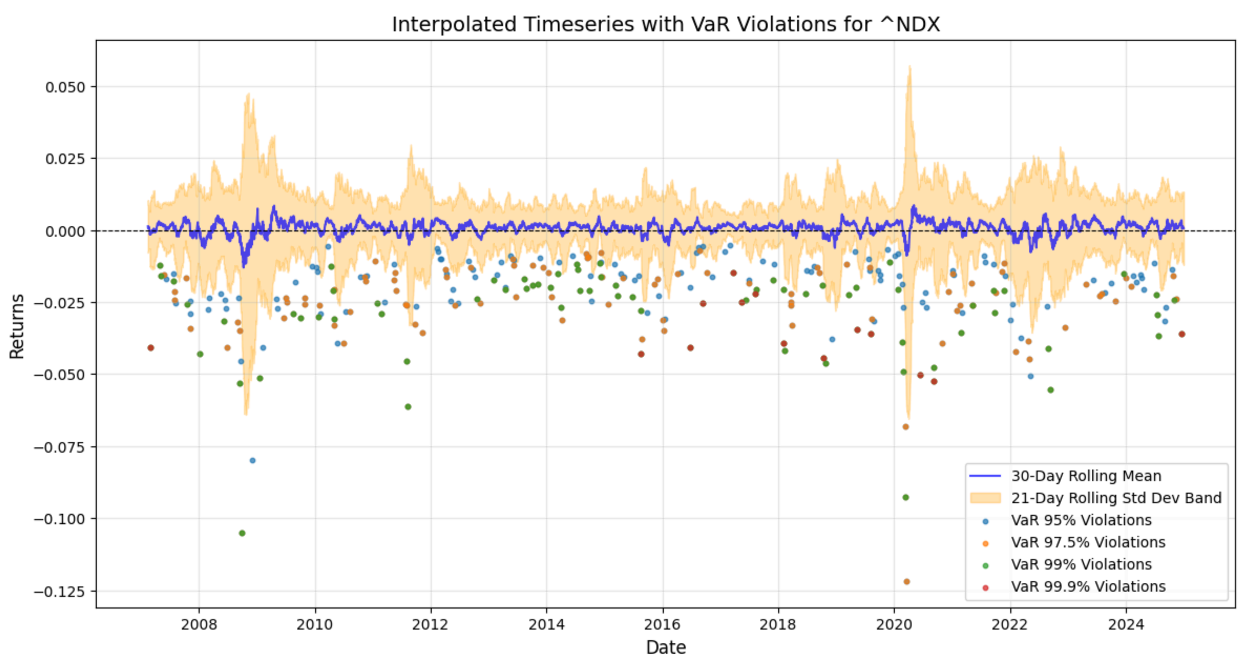
Time-series with VaR violations

The interpolated time-series with VaR violations provides a visual representation of returns over time, along with statistical measures and VaR violations. Each chart includes: 30-Day Rolling Mean, 21-Day Rolling Standard Deviation Band, VaR Violations. All charts display periods of significant volatility, particularly during the GFC and the 2020 COVID-19 market crash. Clustering of VaR violations during these periods indicates extreme market stress. The NASDAQ100 shows higher volatility compared to the other indices with more frequent and severe VaR violations.


Interpolated Timeseries with VaR Violations for ^DJI
Returns
Date
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−0.125
2008
2010
2012
2014
2016
2018
2020
2022
2024
30-Day Rolling Mean
21-Day Rolling Std Dev Band
VaR 95% Violations
VaR 97.5% Violations
VaR 99% Violations
VaR 99.9% Violations


Interpolated Timeseries with VaR Violations for ^GSPC
Returns
Date
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−0.125
2008
2010
2012
2014
2016
2018
2020
2022
2024
30-Day Rolling Mean
21-Day Rolling Std Dev Band
VaR 95% Violations
VaR 97.5% Violations
VaR 99% Violations
VaR 99.9% Violations





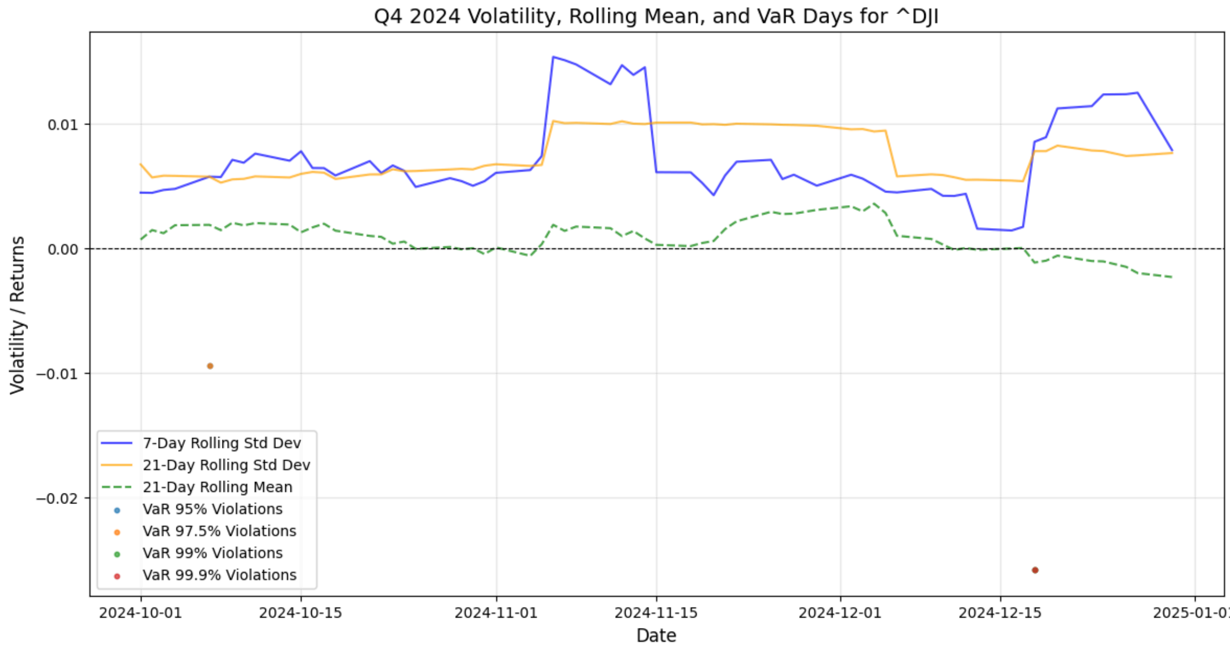
Narrowing the observation timeframe

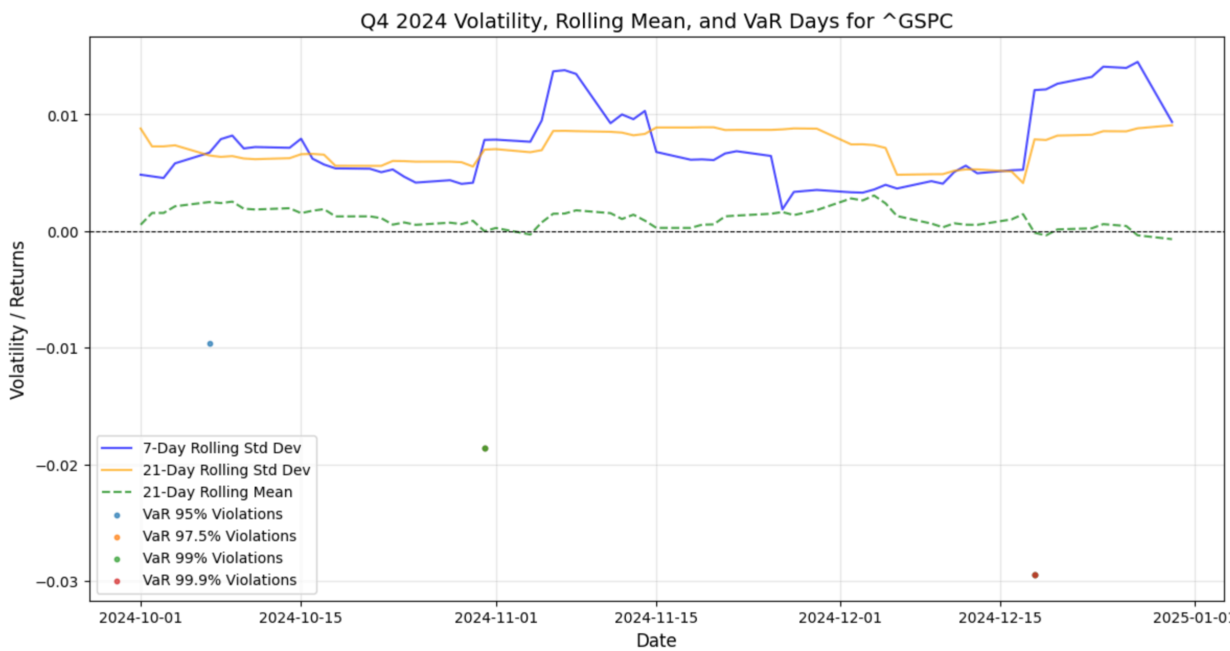
The VaR violations data for 2024 and Q4 2024, along with the ratio of Q4 violations to full-year violations, provides insights into the frequency and concentration of extreme losses across the indices: Dow Jones (^DJI), S&P 500 (^GSPC), NASDAQ 100 (^NDX), and Russell 3000 (^RUA).

Table: VaR Violations in 2024 and Q4 2024

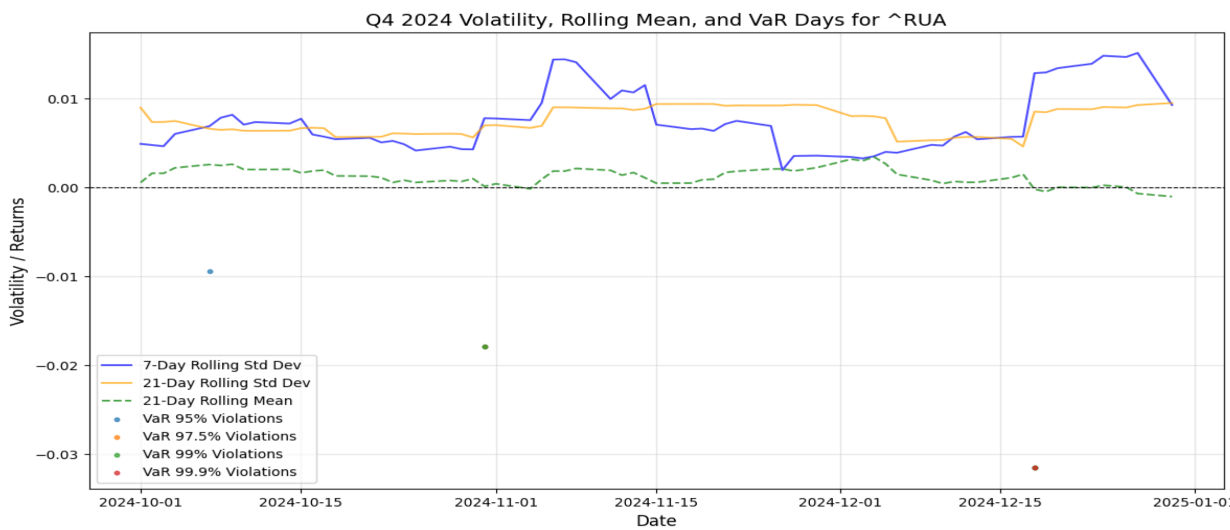
Index	Confidence Level	2024 Violations	Q4 2024 Violations	Ratio (Q4/2024)
^DJI	95%	16	3	18.75%
	97.5%	6	1	16.67%
	99%	2	1	50.00%
	99.9%	1	1	100.00%
^GSPC	95%	14	3	21.43%
	97.5%	10	2	20.00%
	99%	6	2	33.33%
	99.9%	1	1	100.00%
^NDX	95%	18	5	27.78%
	97.5%	9	4	44.44%
	99%	5	2	40.00%
	99.9%	1	1	100.00%
^RUA	95%	17	4	23.53%
	97.5%	9	2	22.22%
	99%	6	2	33.33%
	99.9%	1	1	100.00%

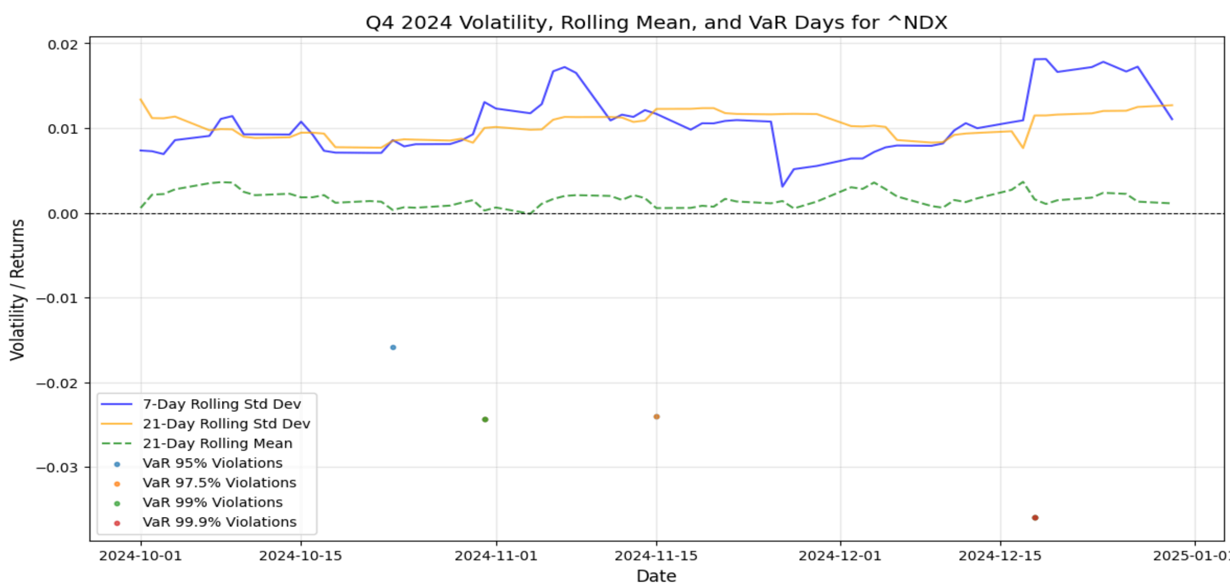
The Q4 2024 volatility, rolling mean, and VaR violation charts for the indices—Dow Jones (^DJI), S&P 500 (^GSPC), NASDAQ 100 (^NDX), and Russell 3000 (^RUA)—provide a detailed view of market behaviour during this period, highlighting the narrowing of the timeframe to identify clustering patterns in VaR violations. These charts illustrate the interplay between volatility, rolling means, and extreme losses (VaR violations) in Q4 2024, offering insights into how market stress manifests over shorter time horizons.





The 7-day and 21-day rolling standard deviations show fluctuations in volatility throughout Q4 2024. Periods of heightened volatility are often accompanied by clusters of VaR violations. The ***NASDAQ 100*** *exhibits higher volatility compared to the other indices, with more pronounced spikes in the 7-day and 21-day rolling standard deviations, as the VaR violations are more frequent and clustered in Q4, reflecting broader market sensitivity to market volatility. In the* ***S&P 500*** *and* ***Russell 3000****, the rolling mean shows a gradual decline in Q4 2024, reflecting* *broader market uncertainty during this period. In the **Dow Jones**, VaR violations are concentrated in mid-October and late December, corresponding to spikes in volatility. The **Q4 2024 charts** demonstrate the value of narrowing the timeframe to identify **clustering patterns** in VaR violations. These patterns are closely tied to periods of heightened volatility and market stress, with the **NASDAQ 100** showing the most frequent and severe clustering due to its higher sensitivity to market shocks. The **Dow Jones**, with its stable composition, exhibits fewer and less severe clusters, while the **S&P 500** and **Russell 3000** show moderate clustering, reflecting their diversified risk profiles. Understanding these patterns is crucial for forecasting market drawdowns and implementing effective risk management strategies.*





Implementing GARCH and EGARCH Models

This section presents the implementation of GARCH (Generalized Autoregressive Conditional Heteroskedasticity) and EGARCH (Exponential GARCH) models to analyze the volatility dynamics of the Dow Jones Industrial Average (^DJI). The results from these models provide insights into the persistence of volatility, the impact of past shocks, and the presence of asymmetric effects (leverage effects) in the index.

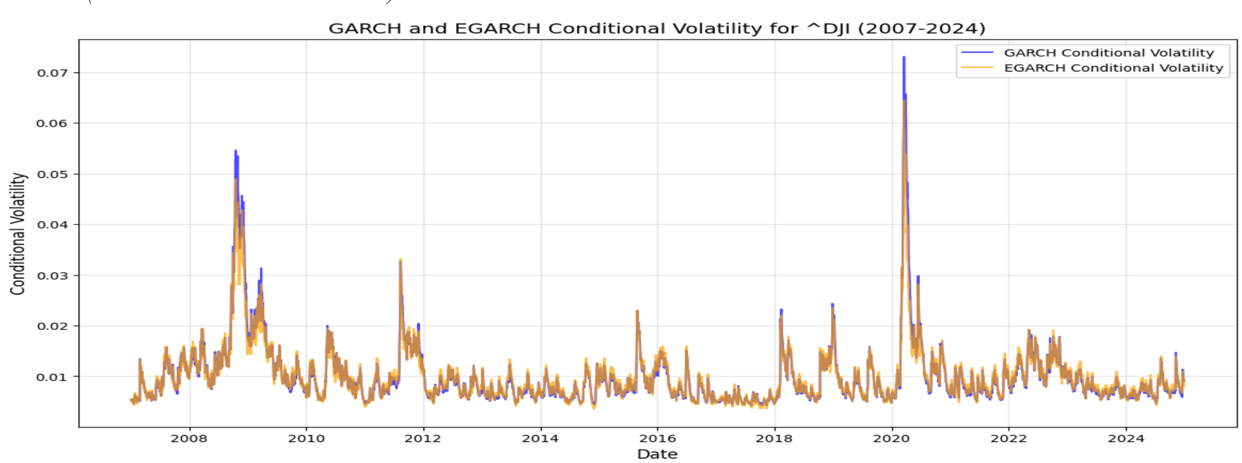
$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2$$

$$\log(\sigma_t^2) = \omega + \alpha\left(\frac{|\epsilon_{t-1}|}{\sigma_{t-1}}\right) + \gamma\frac{\epsilon_{t-1}}{\sigma_{t-1}} + \beta\log(\sigma_{t-1}^2)$$

*The **GARCH model** captures the time-varying volatility of the Dow Jones, with the following key findings:*

***Mean Return (mu)**: The model estimates a small but statistically significant ($p<0.001$) average daily return of 0.0677%, while the alpha (0.100) parameter indicates that 10% of the previous day's square returns contribute to volatility. The beta terms (0.880) indicate that 88% of the previous day's volatility persists across time. The GARCH model provides a good fit, with a log-likelihood of (12718.8 and low AIC/BIC values (−25429.7) and (−25404.7) respectively.*

The EGARCH model extends the GARCH framework by incorporating asymmetric effects, in fact, the model estimates a slightly higher average daily return of 0.085179%, statistically significant $p<0.001$). The Alpha term (0.3194) implies that 31.94% of past shocks have a stronger correlation with current volatility and the Beta (0.9505) indicates even stronger volatility persistence of 95.05% than the GARCH model. The EGARCH model indeed provides a better fit than the GARCH model, with a higher log-likelihood (1273.10) and lower AIC/BIC values (-25454.1 and -25429.1)



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GARCH Model Summary for ^DJI:
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   ^DJI   R-squared:                       0.00
Mean Model:             Constant Mean   Adj. R-squared:                  0.00
Vol Model:                      GARCH   Log-Likelihood:                 12718.
Distribution:                  Normal   AIC:                           -25429.
Method:            Maximum Likelihood   BIC:                           -25404.
                                        No. Observations:                 3772
Date:                Tue, Dec 31 2024   Df Residuals:                     3771
Time:                        14:28:18   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         6.7744e-04  1.112e-04      6.094  1.103e-09 [4.596e-04,8.953e-04]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      2.1873e-06  2.207e-13  9.911e+06      0.000 [2.187e-06,2.187e-06]
alpha[1]       0.1000  1.650e-04    606.052      0.000   [9.968e-02,  0.100]
beta[1]        0.8800  3.228e-03    272.606      0.000     [  0.874,  0.886]
============================================================================

Covariance estimator: robust
EGARCH Model Summary for ^DJI:
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                   ^DJI   R-squared:                       0.00
Mean Model:             Constant Mean   Adj. R-squared:                  0.00
Vol Model:                     EGARCH   Log-Likelihood:                 12731.
Distribution:                  Normal   AIC:                           -25454.
Method:            Maximum Likelihood   BIC:                           -25429.
                                        No. Observations:                 3772
Date:                Tue, Dec 31 2024   Df Residuals:                     3771
Time:                        14:28:18   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         8.5179e-04  1.244e-04      6.845  7.648e-12 [6.079e-04,1.096e-03]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega         -0.4573  8.109e-02     -5.639  1.709e-08 [ -0.616, -0.298]
alpha[1]       0.3194  3.020e-02     10.578  3.768e-26 [  0.260,  0.379]
beta[1]        0.9505  8.621e-03    110.260      0.000 [  0.934,  0.967]
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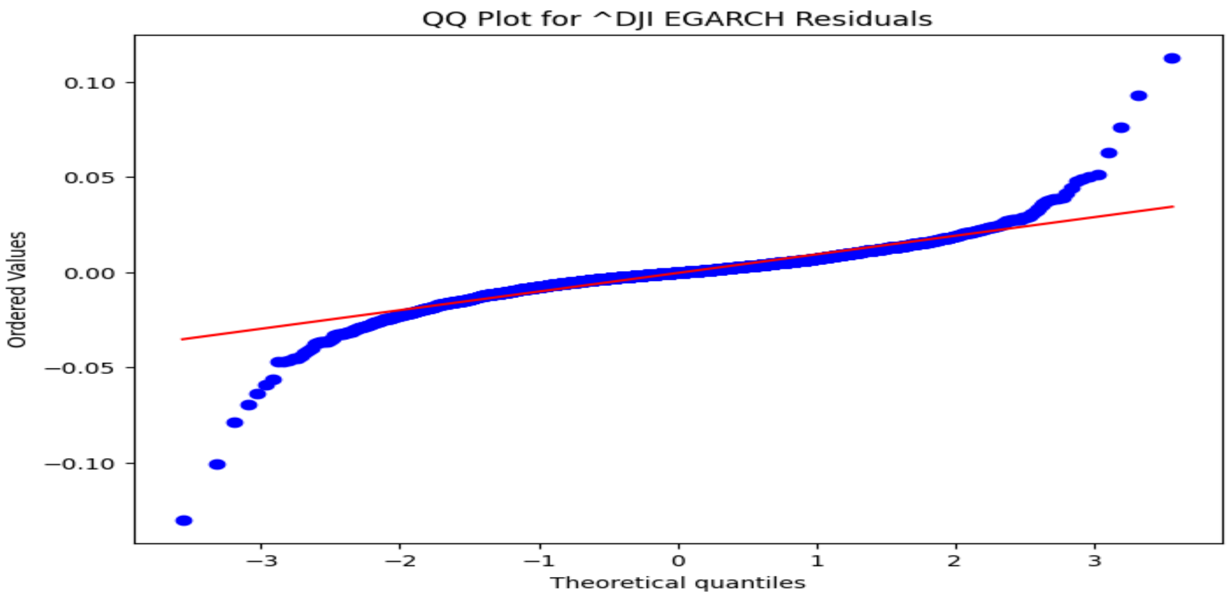
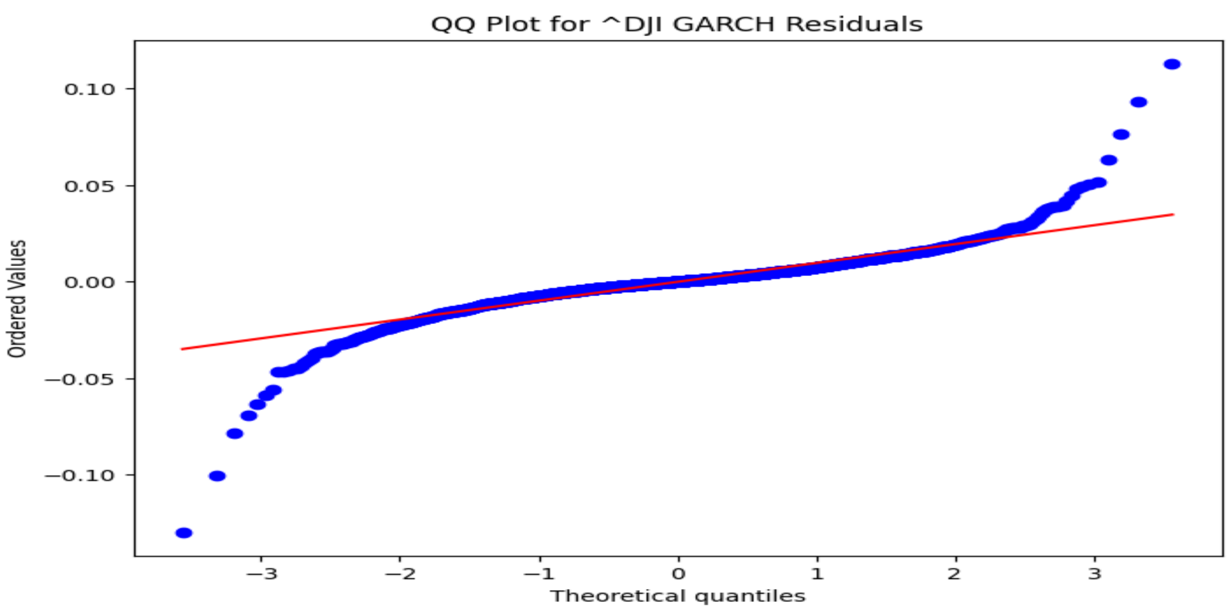
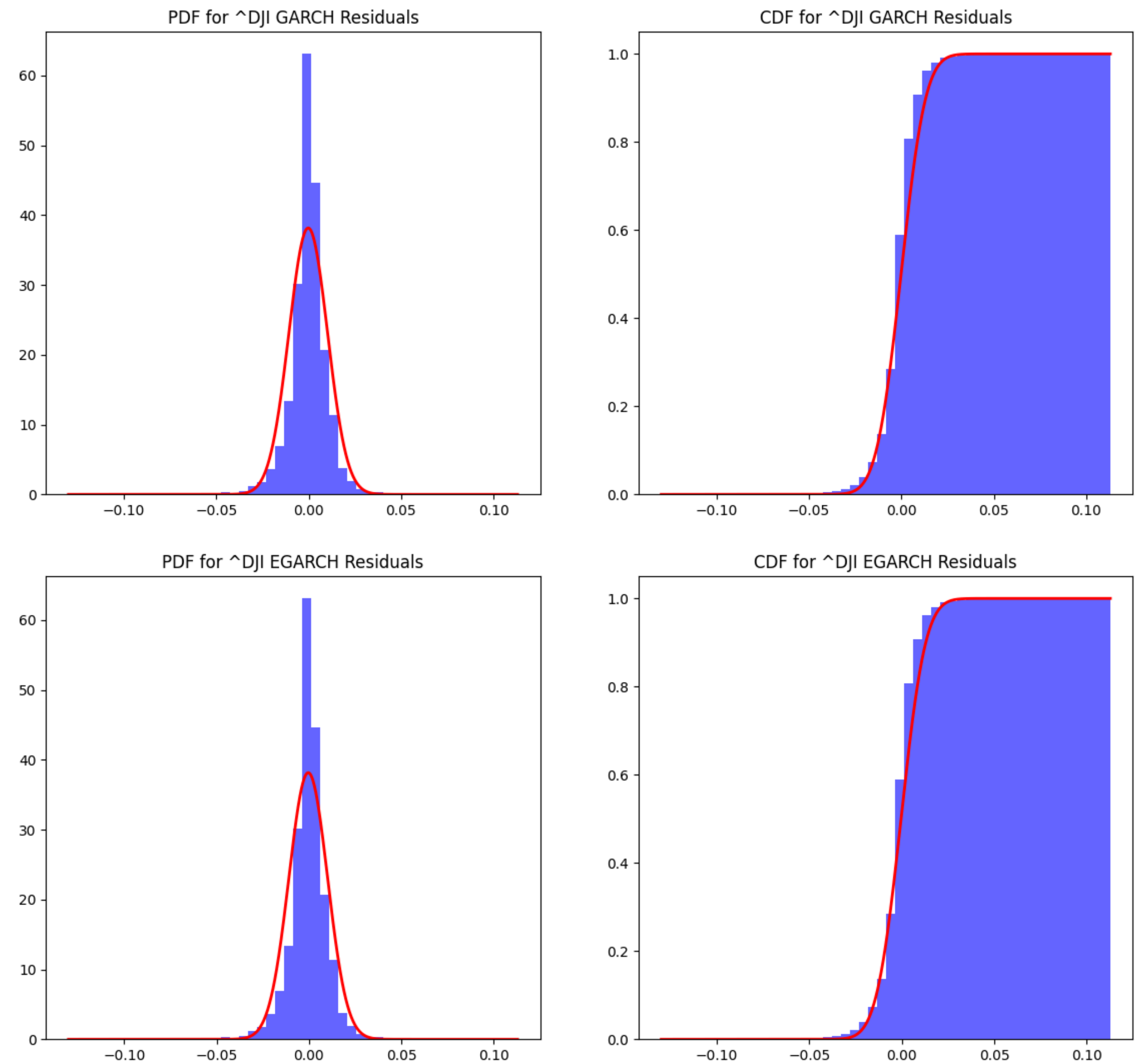




*Both the **GARCH** and **EGARCH** QQ plots show deviations from the theoretical normal distribution, particularly in the tails. The residuals exhibit **heavier tails** than a normal distribution, indicating the presence of extreme values that the models do not fully capture.*



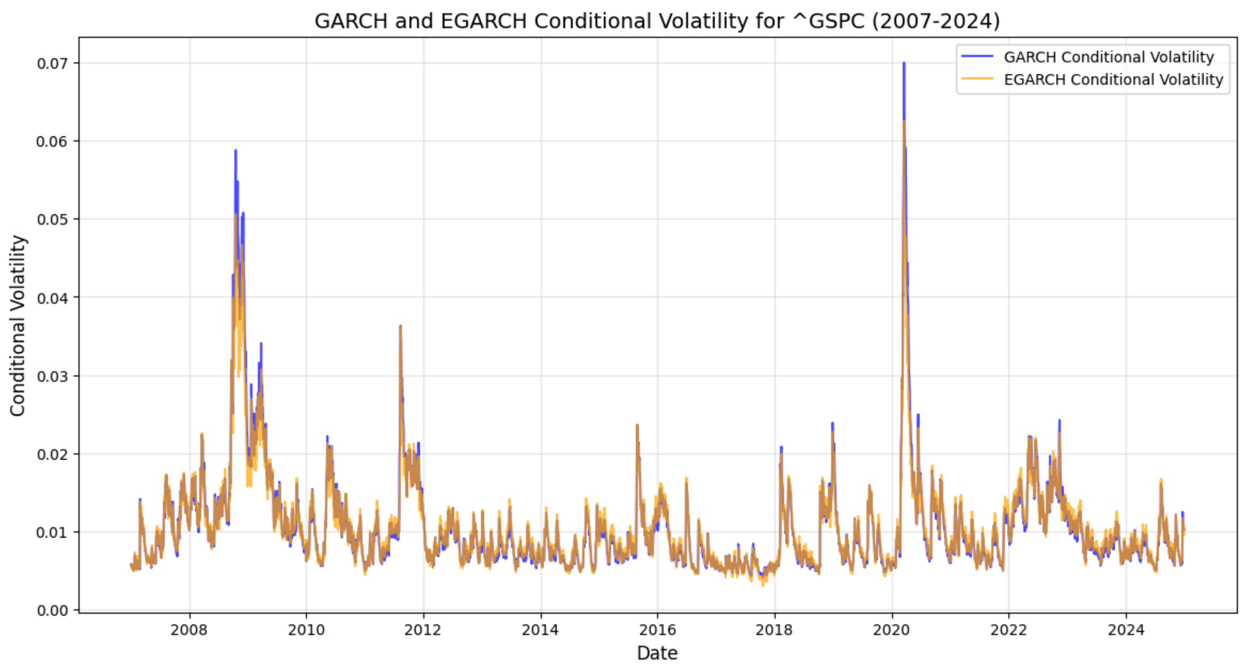
Both the GARCH and EGARCH residual distributions are leptokurtic (peaked with heavy tails), which is typical for financial data. The EGARCH residuals appear slightly better behaved, with a more symmetric distribution around zero, reflecting its ability to capture asymmetric effects. However, both models still show deviations from normality, particularly in the tails, reinforcing the need for robust risk management techniques like Extreme Value Theory (EVT).

GARCH and EGARCH model summaries for the S&P 500 (^GSPC)

The GARCH model results for all four indices display the ***same parameter values*** *for the volatility equation: Omega (baseline volatility): DJI (0.0000021873), GSPC Omega (0.0000023570), NDX Omega (0.0000023000), RUA Omega (0.0000024000). For all four*

GARCH model index results the Alpha parameter equates (0.100) indicating that 10% of the previous day's squared returns contribute to volatility across time, the Alpha parameter is statistically significant (p<0.001) indeed proving that past shocks have a moderate impact on current volatility. The Beta parameter for all four indexes equated to (0.88) proving that 88% of the previous day's volatility persists across the time-series and the parameter is highly significant (p<0.001) showing strong volatility clustering.

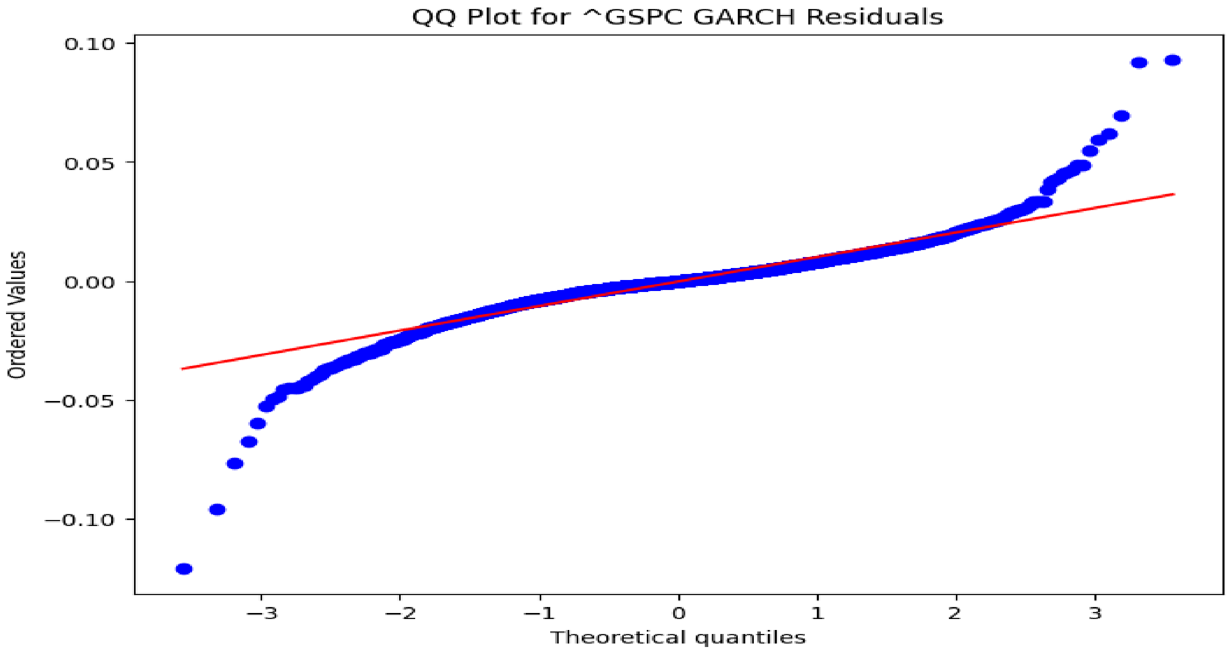
The S&P500(GSPC) EGARCH model displays specific results with a (mu) parameter of (0.0091) and the parameter is highly significant (p<0.001), while the Omega (-0.4569) highly significant (p<0.001) represents the baseline level of volatility in the logarithmic form. The Alpha parameter (0.3176) indicates that past shocks have a stronger impact on current volatility compared to GARCH models, with statistically significant (p<0.001). The Beta parameter (0.9500) indicates that 95% of the previous day's volatility persists across time, also statistically significant (p<0.001) showing even stronger volatility clustering than the GARCH model.

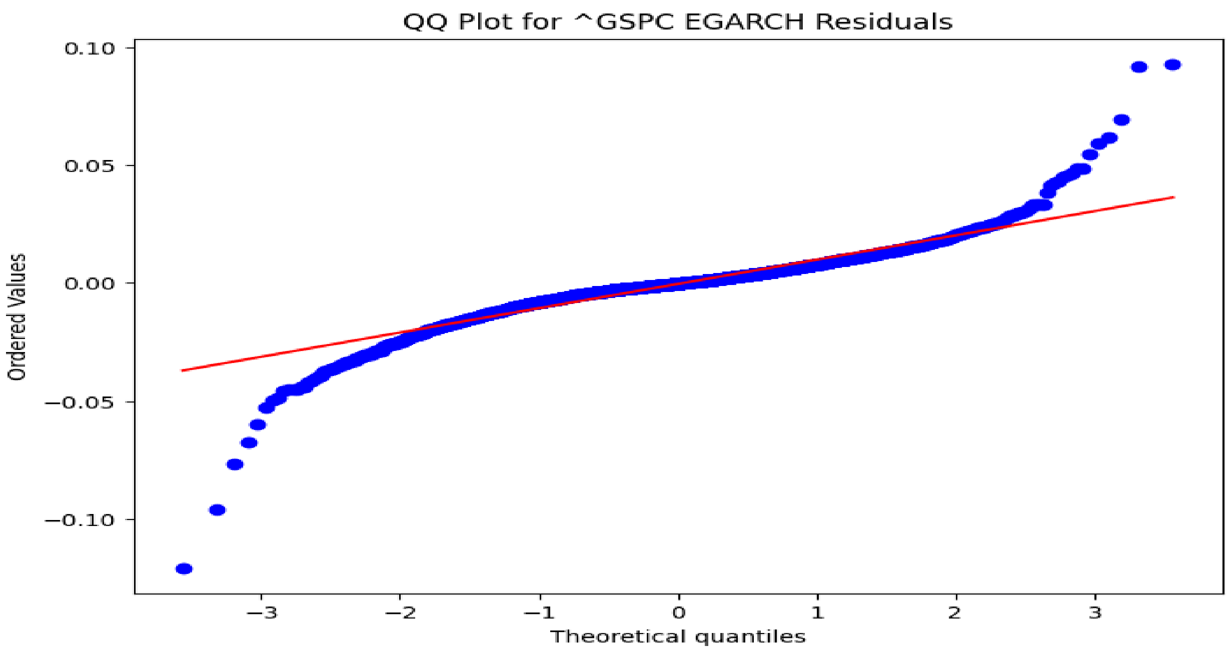


The EGARCH model provides a better fit than the GARCH model, as indicated by the higher log-likelihood and lower AIC/BIC values. This suggests that the EGARCH model is more appropriate for modelling the S&P 500's volatility, especially when accounting for asymmetric effects.

```
GARCH Model Summary for ^GSPC:
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                  ^GSPC   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                12514.4
Distribution:                  Normal   AIC:                          -25020.8
Method:            Maximum Likelihood   BIC:                          -24995.8
                                        No. Observations:                 3772
Date:                Tue, Dec 31 2024   Df Residuals:                     3771
Time:                        14:28:21   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         7.5394e-04  4.385e-05     17.195  2.899e-66 [6.680e-04,8.399e-04]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      2.3570e-06  2.724e-12  8.653e+05      0.000 [2.357e-06,2.357e-06]
alpha[1]       0.1000  1.181e-02      8.465  2.570e-17   [7.685e-02,  0.123]
beta[1]        0.8800  9.402e-03     93.597      0.000     [  0.862,  0.898]
============================================================================

Covariance estimator: robust
EGARCH Model Summary for ^GSPC:
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                  ^GSPC   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:                12521.5
Distribution:                  Normal   AIC:                          -25035.0
Method:            Maximum Likelihood   BIC:                          -25010.1
                                        No. Observations:                 3772
Date:                Tue, Dec 31 2024   Df Residuals:                     3771
Time:                        14:28:21   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         9.1100e-04  1.258e-04      7.241  4.453e-13 [6.644e-04,1.158e-03]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega         -0.4569  8.744e-02     -5.225  1.745e-07 [ -0.628, -0.285]
alpha[1]       0.3176  3.136e-02     10.129  4.117e-24 [  0.256,  0.379]
beta[1]        0.9500  9.476e-03    100.256      0.000 [  0.931,  0.969]
========================================================================
```

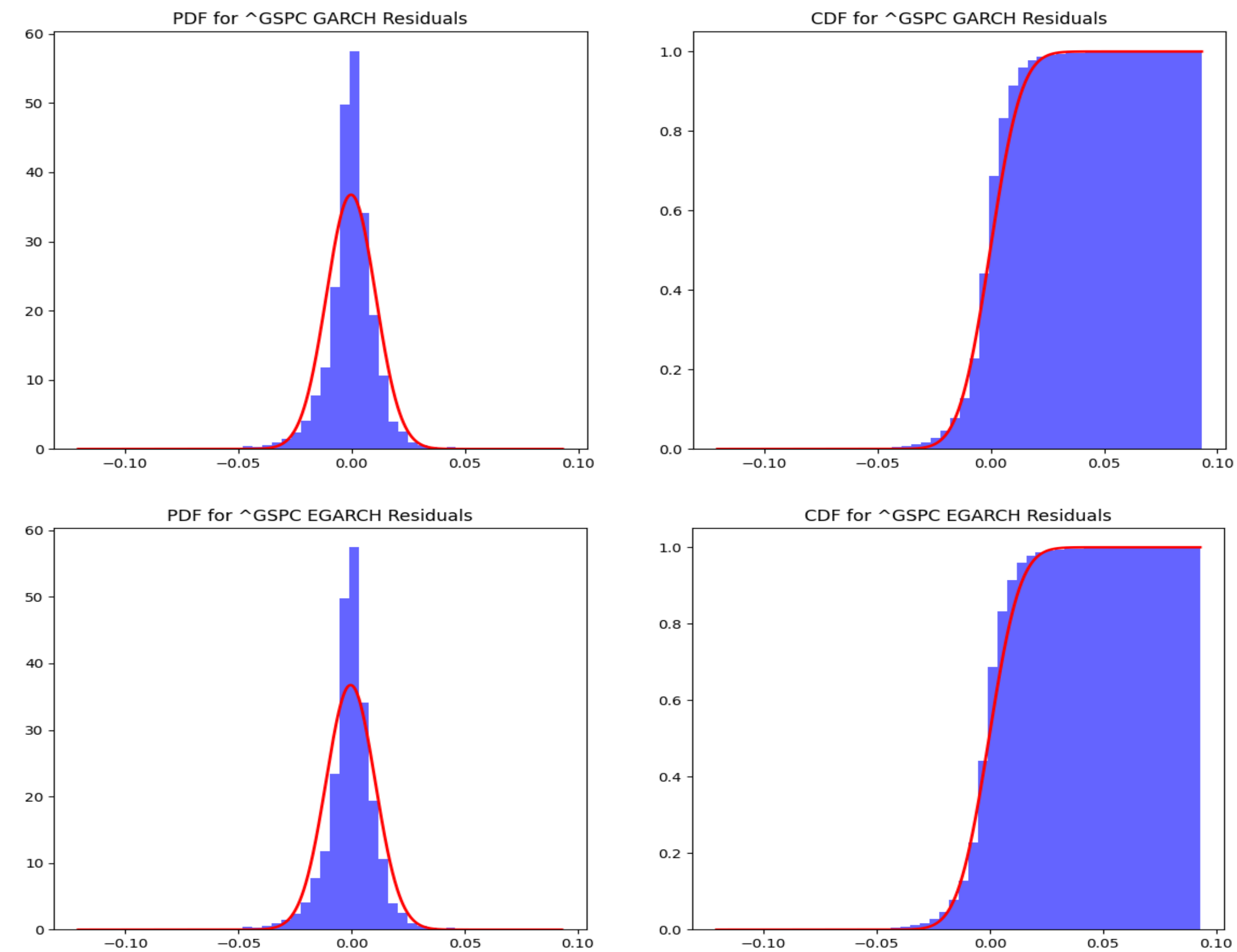
The QQ plots compare the distribution of the model residuals to a normal distribution, revealing heavy tails in both the GARCH and EGARCH residuals. This indicates that the models underestimate the likelihood of extreme events, a common limitation in financial data modelling. However, the EGARCH residuals appear slightly better behaved, with a more symmetric distribution around zero, reflecting its ability to capture asymmetric effects (leverage effects).



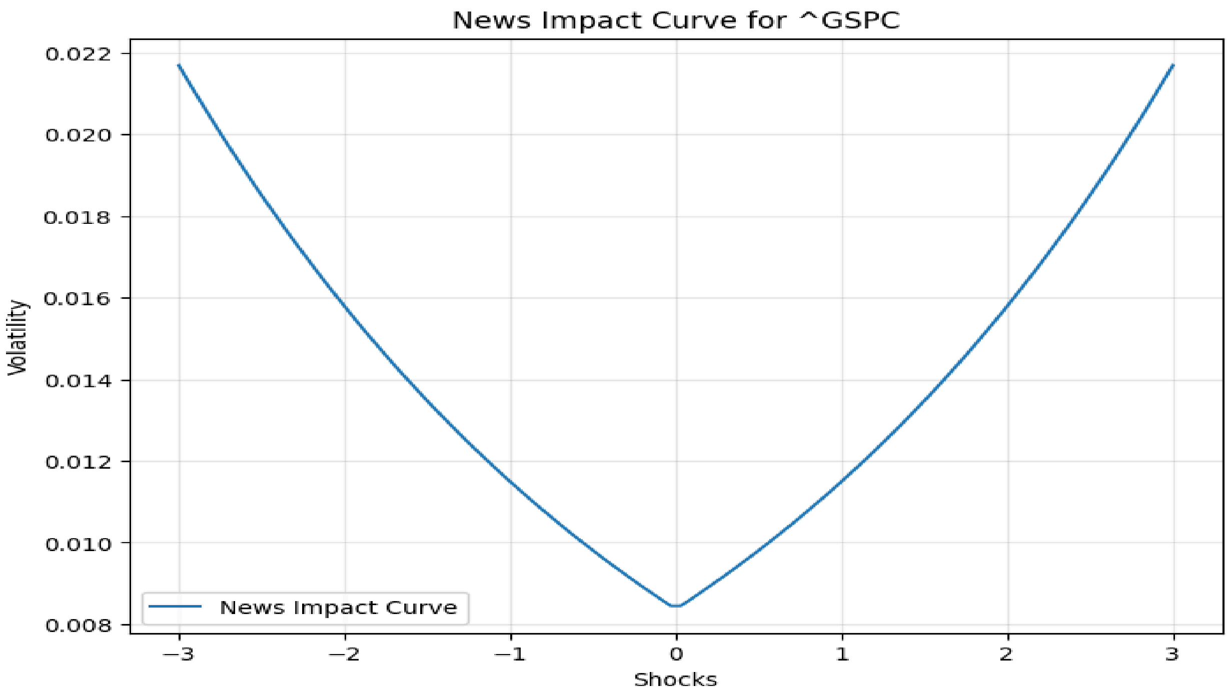


The PDF (Probability Density Function) and CDF (Cumulative Distribution Function) plots for the GARCH and EGARCH residuals of the S&P 500 (^GSPC) provide further insights into the *distribution of model residuals. The PDFs for both models show that the residuals are leptokurtic, meaning they have heavier tails and a sharper peak compared to a normal distribution. This is typical for financial data, where extreme events occur more frequently than predicted by a normal distribution. The EGARCH residuals appear slightly more symmetric around zero, reflecting the model's ability to better capture asymmetric effects (leverage effects).*

The CDFs illustrate the cumulative probability of the residuals, showing how well the models capture the distribution of returns. While both models exhibit deviations from normality in the tails, the EGARCH model provides a better fit, as evidenced by its more symmetric and centred distribution. This suggests that the EGARCH model is more effective in modelling the S&P 500's volatility dynamics, particularly in capturing extreme events and asymmetric responses to shocks. These findings reinforce the importance of using EGARCH for accurate volatility forecasting and risk management.

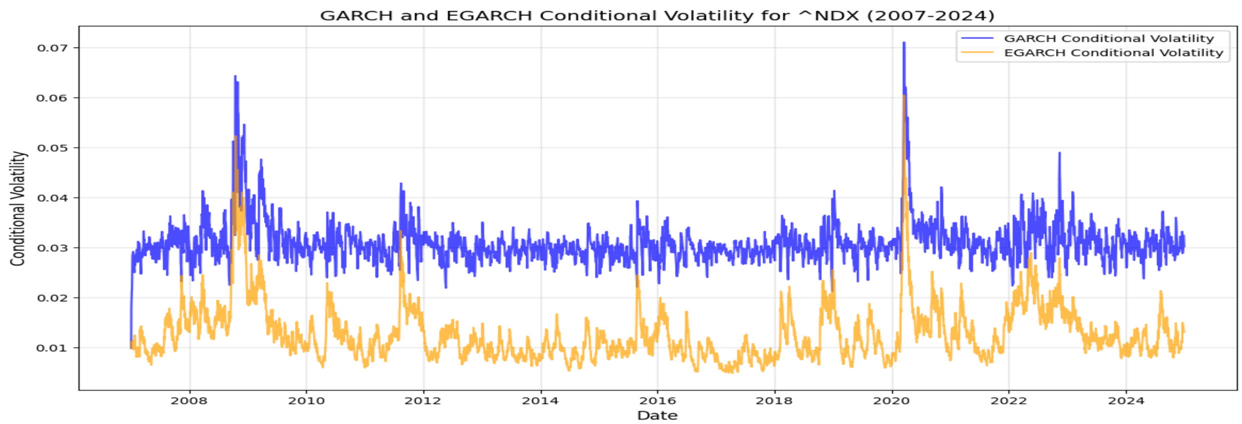


The news impact curve demonstrates how volatility responds to shocks (positive and negative returns). The curve is asymmetric, with negative shocks having a greater impact on volatility than positive shocks of the same magnitude. This aligns with the leverage effect, where bad news increases volatility more than good news. The EGARCH model, with its ability to model this asymmetry, provides a more accurate representation of the S&P 500's volatility dynamics compared to the GARCH model.



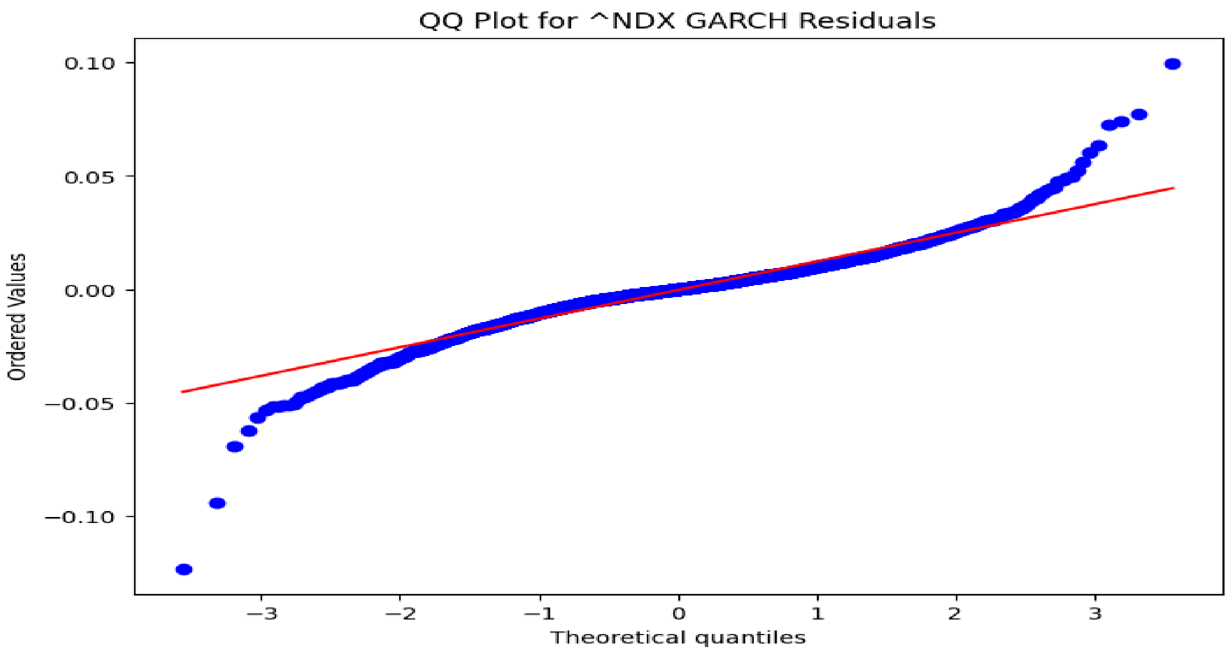
GARCH and EGARCH Model Summary for Nasdaq100

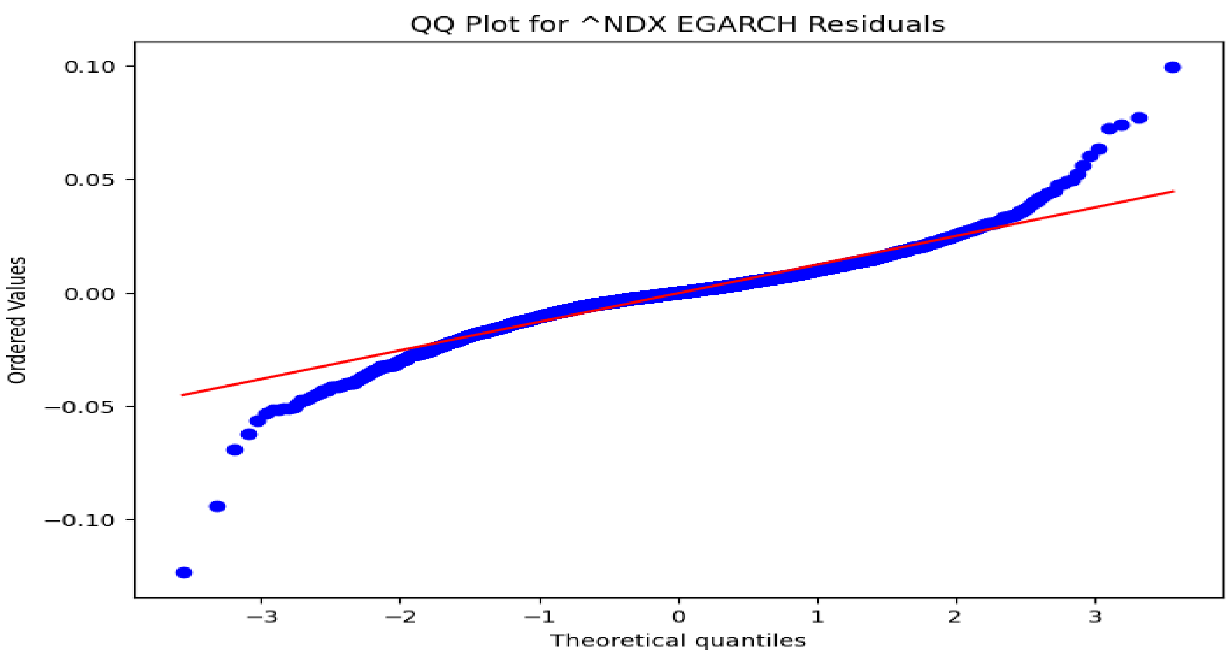
As previously specified the GARCH models display consistently similar parameters across time-series, in this instance, the EGARCH model results for the Nasdaq100 better display the asymmetry in volatility shocks, resulting in parameters: (mu) coefficient 0.00109 statistically significant (p<0.001), the Omega constant (-03773) represents the baseline level of volatility in the logarithmic form. Alpha term (0.2637) indicates that past shocks have a stronger impact on current volatility compared to the GARCH model, also the Beta parameter (0.9566) displays a strong volatility persistency across time, all parameters are highly significant (p<0.001). The ***EGARCH model*** *provides a slightly better fit than the GARCH model, as indicated by the higher log-likelihood and lower AIC/BIC values. This suggests that the EGARCH model is more appropriate for modelling the NASDAQ 100's volatility, especially when accounting for* *asymmetric effects.*



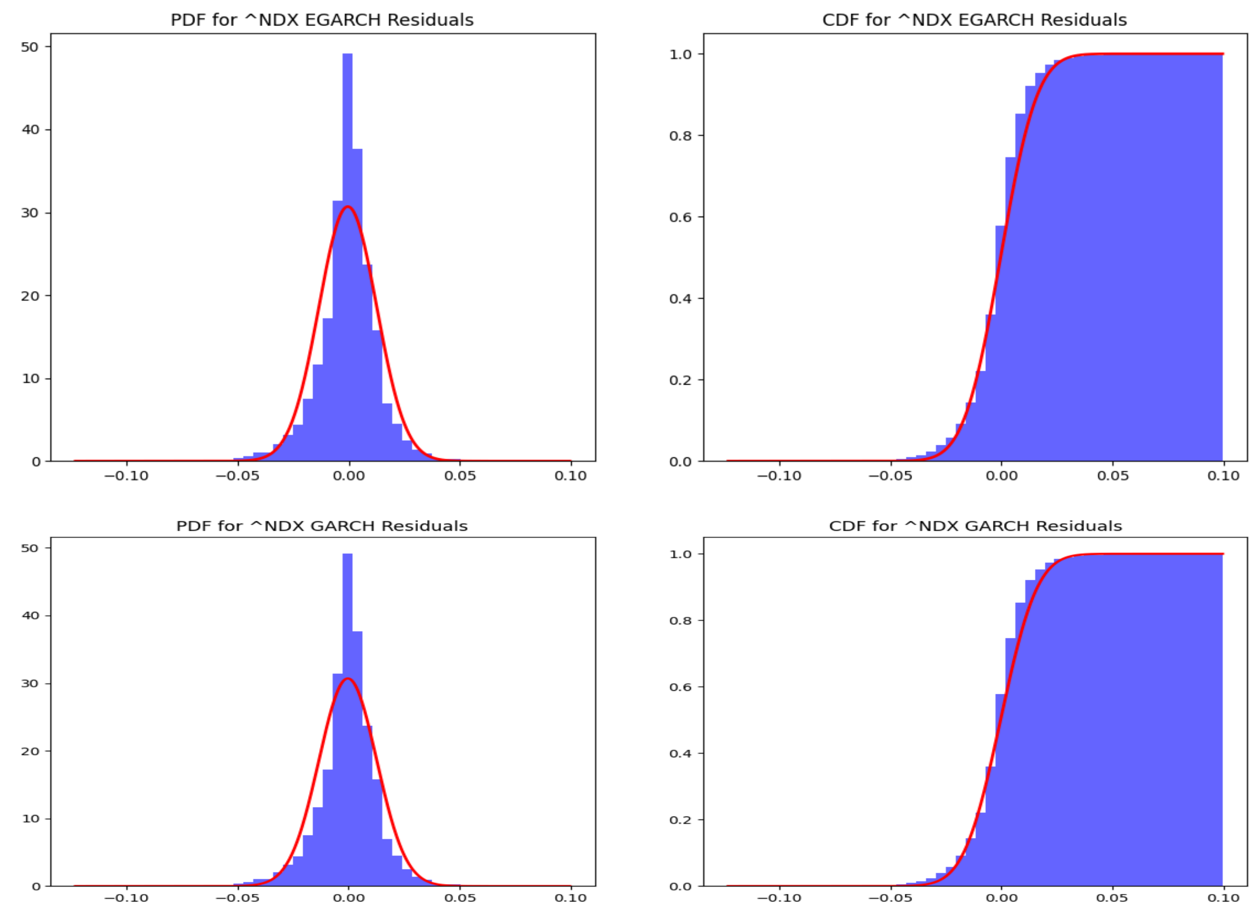

```
GARCH Model Summary for ^NDX:
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   ^NDX   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                11628.1
Distribution:                  Normal   AIC:                          -23248.2
Method:            Maximum Likelihood   BIC:                          -23223.3
                                        No. Observations:                 3772
Date:                Tue, Dec 31 2024   Df Residuals:                     3771
Time:                        14:28:23   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         1.0552e-03  1.633e-04      6.463  1.024e-10 [7.352e-04,1.375e-03]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      3.3802e-06  5.803e-13  5.825e+06      0.000 [3.380e-06,3.380e-06]
alpha[1]       0.1000  5.801e-04    172.382      0.000   [9.886e-02,  0.101]
beta[1]        0.8800  3.619e-03    243.133      0.000     [  0.873,  0.887]
============================================================================

Covariance estimator: robust
EGARCH Model Summary for ^NDX:
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                   ^NDX   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:                11627.7
Distribution:                  Normal   AIC:                          -23247.4
Method:            Maximum Likelihood   BIC:                          -23222.4
                                        No. Observations:                 3772
Date:                Tue, Dec 31 2024   Df Residuals:                     3771
Time:                        14:28:23   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         1.1099e-03  1.370e-04      8.102  5.425e-16 [8.414e-04,1.378e-03]
                            Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega         -0.3773  7.295e-02     -5.172  2.317e-07 [ -0.520, -0.234]
alpha[1]       0.2637  2.545e-02     10.361  3.735e-25 [  0.214,  0.314]
beta[1]        0.9566  8.263e-03    115.778      0.000 [  0.940,  0.973]
========================================================================
```

The QQ plots for the GARCH and EGARCH residuals of the NASDAQ 100 (^NDX) compare the distribution of residuals to a normal distribution. Both plots show heavy tails, indicating that the residuals have more extreme values than expected under a normal distribution. This is typical for financial data, where extreme events occur more frequently. The EGARCH residuals appear slightly more symmetric around zero, reflecting the model's ability to better capture asymmetric effects (leverage effects). However, both models still exhibit deviations from normality, particularly in the tails, highlighting the need for robust risk management techniques like Extreme Value Theory (EVT) to account for tail risk.

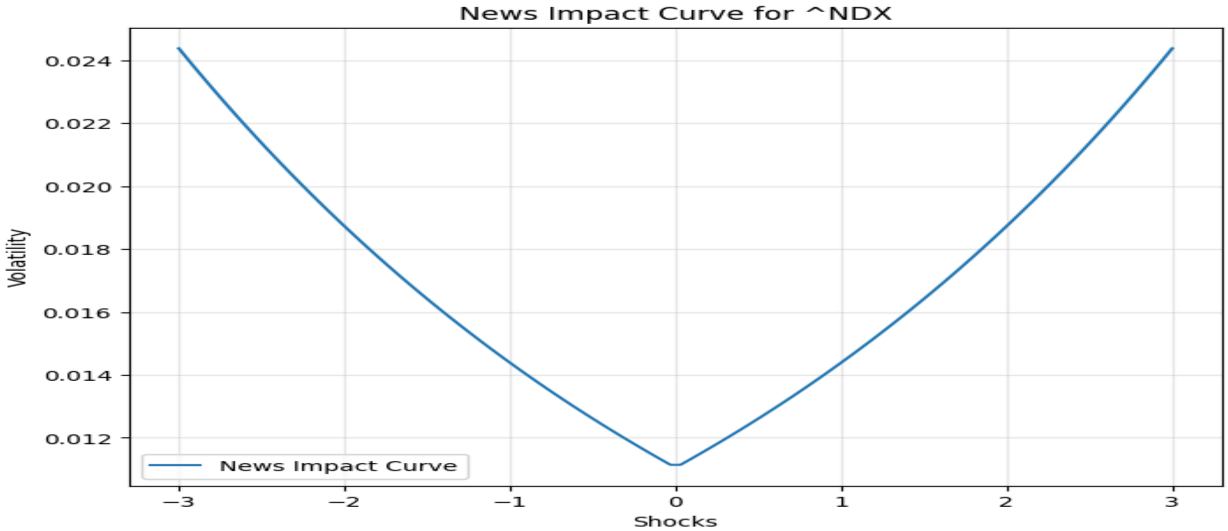




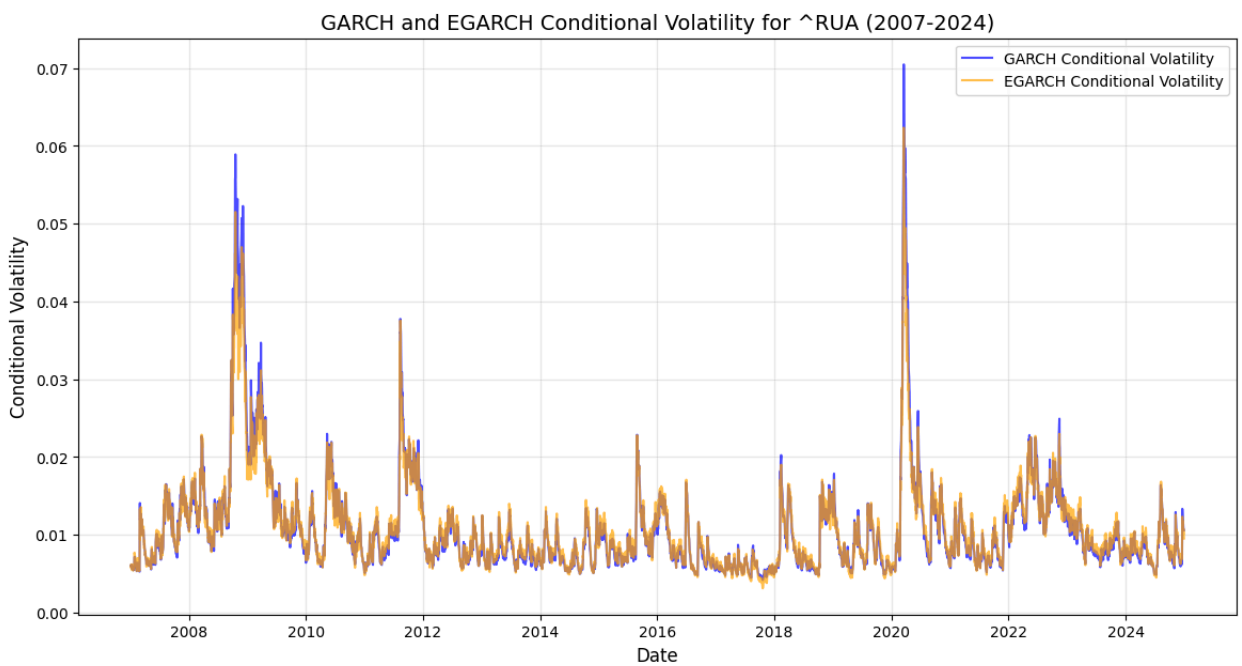
The PDFs (Probability Density Functions) for the GARCH and EGARCH residuals show that the residuals are leptokurtic, meaning they have heavier tails and a sharper peak compared to a normal distribution. This is consistent with the presence of extreme events in financial data. The EGARCH residuals are slightly more symmetric and centred around zero, reflecting the model's ability to better capture asymmetric effects. This suggests that the EGARCH model provides a more accurate representation of the NASDAQ 100's volatility dynamics, particularly in capturing extreme events and asymmetric responses to shocks. The CDFs (Cumulative Distribution Functions) for the residuals illustrate the cumulative probability of the residuals, showing how well the models capture the distribution of returns. Both models exhibit deviations from normality in the tails, but the EGARCH model provides a better fit, as evidenced by its more symmetric and centred distribution. This reinforces the importance of using EGARCH for accurate volatility forecasting and risk management, particularly for indices like the NASDAQ 100 that are prone to extreme events and asymmetric volatility responses.



The news impact curve illustrates how volatility responds to shocks (positive and negative returns) for the NASDAQ 100. The curve is asymmetric, with negative shocks having a greater impact on volatility than positive shocks of the same magnitude. This aligns with the leverage effect, where bad news increases volatility more than good news. The EGARCH model, which *explicitly accounts for this asymmetry, provides a more accurate representation of the NASDAQ 100's volatility dynamics compared to the GARCH model. This finding underscores the importance of using EGARCH for modelling indices with significant leverage effects, such as the technology-heavy NASDAQ 100.*

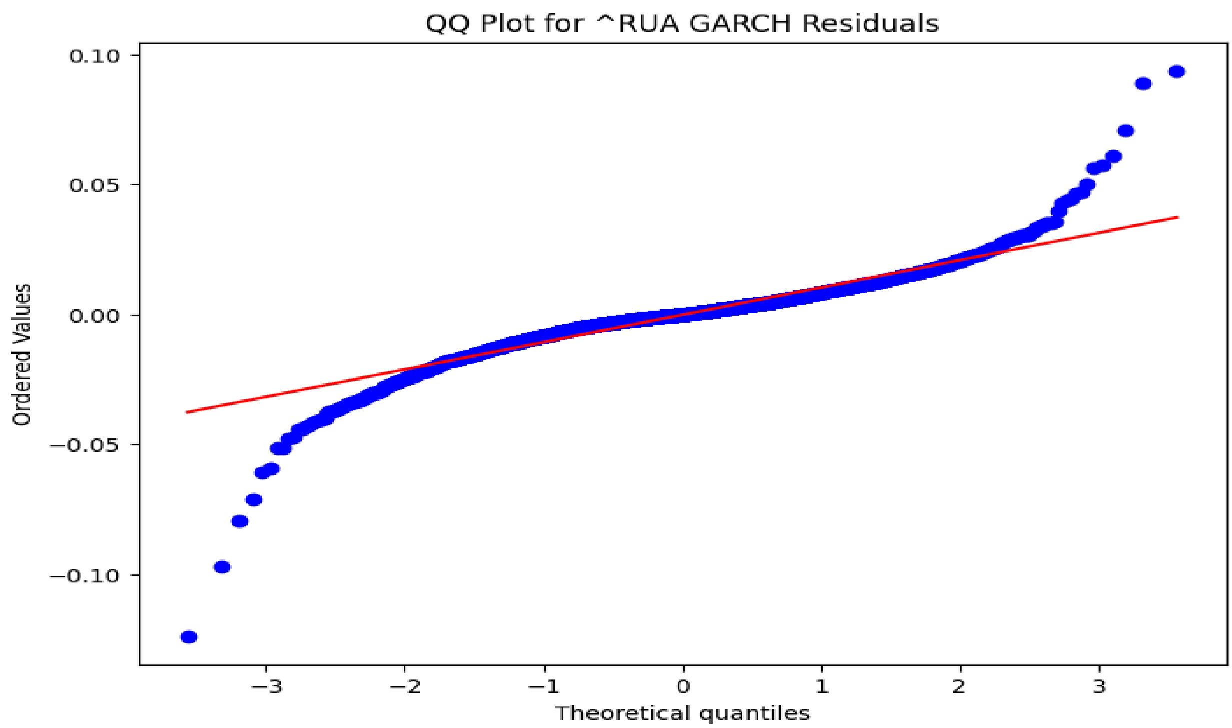


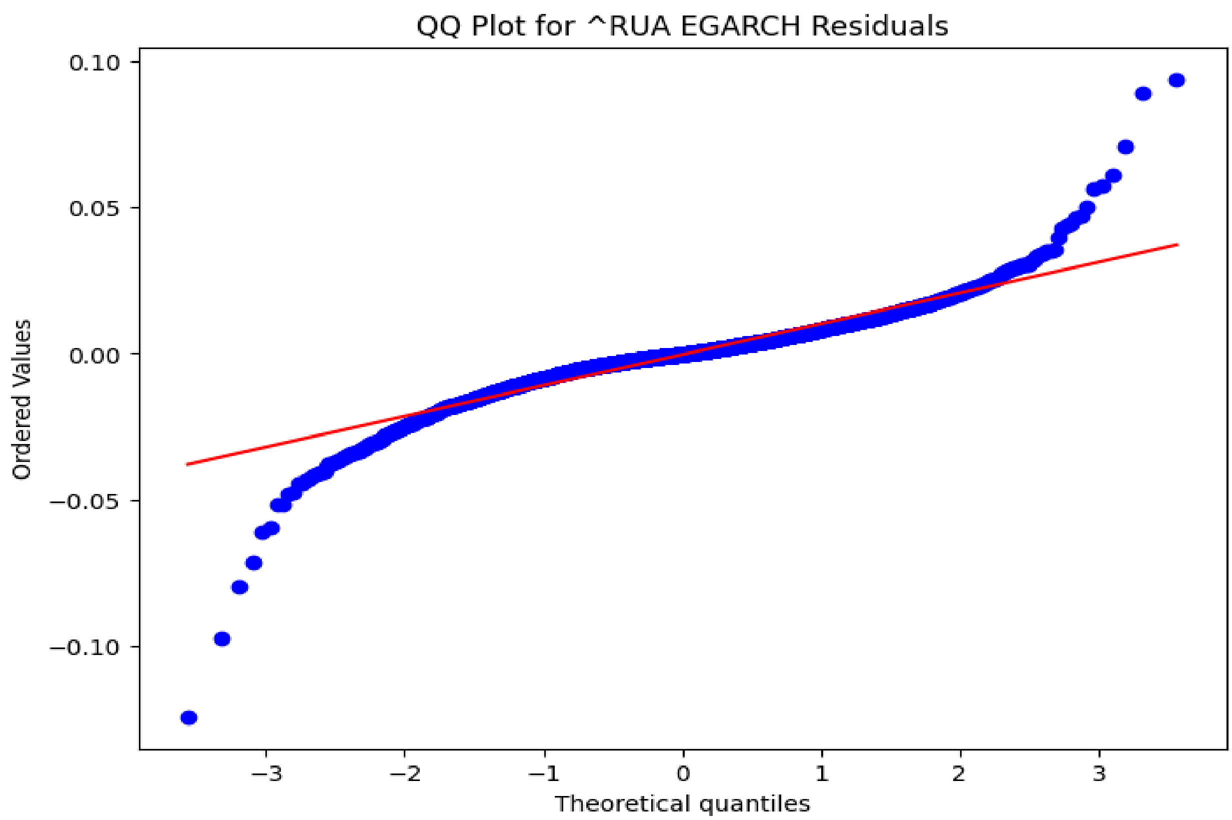
GARCH and EGARCH model results of the Russell 3000

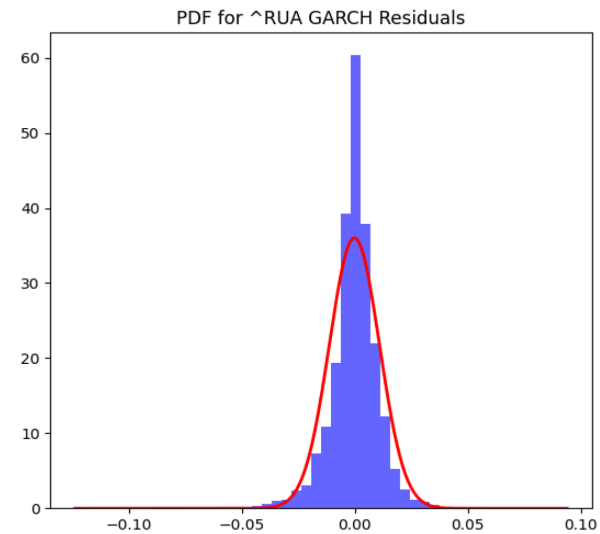


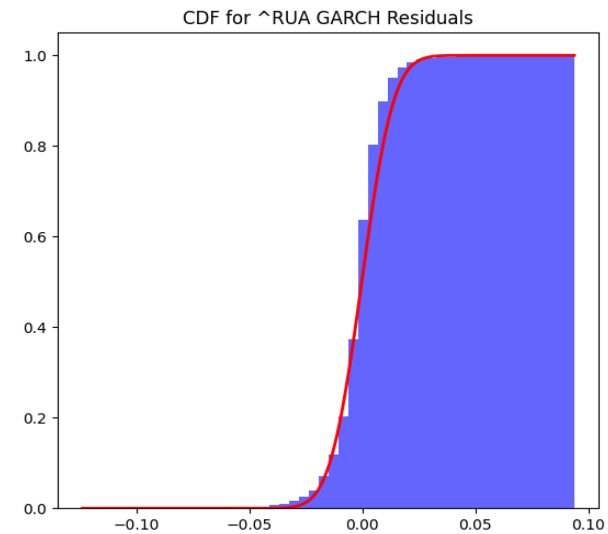
The EGARCH model results for the Russell 3000 display a (mu) coefficient (0.00089127), and an Omega constant (-0.4071) as baseline volatility in logarithmic form. The Alpah parameter (0.2958) indicates that past shocks have a stronger impact on current volatility. The Beta parameter (0.9551) indicates that 95.51% of the previous volatility persists across time, all terms *highly statistically significant ($p<0.001$). The EGARCH model captures **asymmetric effects** (leverage effects), where negative shocks have a greater impact on volatility than positive shocks. This is reflected in the **alpha[1]** coefficient, which is higher in the EGARCH model. The **EGARCH model** provides a slightly better fit than the GARCH model, as indicated by the higher log-likelihood and lower AIC/BIC values. This suggests that the EGARCH model is more appropriate for modelling the Russell 3000's volatility, especially when accounting for asymmetric effects.*

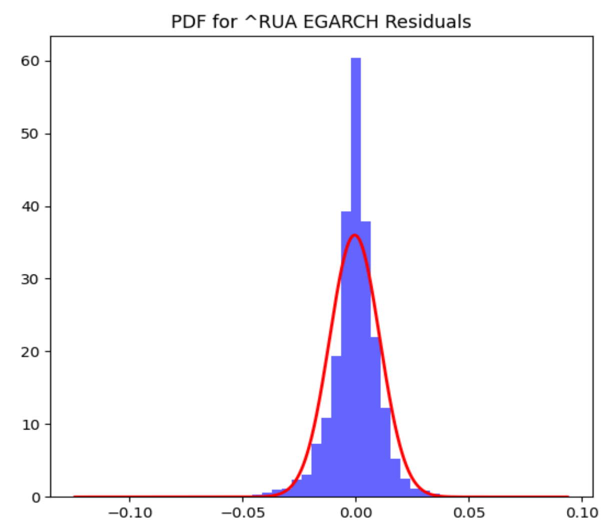
```
GARCH Model Summary for ^RUA:
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                    ^RUA   R-squared:                       0.000
Mean Model:              Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       GARCH   Log-Likelihood:                12420.9
Distribution:                   Normal   AIC:                          -24833.9
Method:             Maximum Likelihood   BIC:                          -24808.9
                                         No. Observations:                 3772
Date:                 Tue, Dec 31 2024   Df Residuals:                     3771
Time:                         14:28:26   Df Model:                            1
                                Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         7.4823e-04  1.224e-04      6.111  9.922e-10 [5.082e-04,9.882e-04]
                              Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      2.4566e-06  1.628e-13  1.509e+07      0.000 [2.457e-06,2.457e-06]
alpha[1]       0.1000  4.125e-04    242.436      0.000   [9.919e-02,  0.101]
beta[1]        0.8800  3.614e-03    243.469      0.000     [  0.873,  0.887]
============================================================================

Covariance estimator: robust
EGARCH Model Summary for ^RUA:
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                    ^RUA   R-squared:                       0.000
Mean Model:              Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      EGARCH   Log-Likelihood:                12420.9
Distribution:                   Normal   AIC:                          -24833.8
Method:             Maximum Likelihood   BIC:                          -24808.8
                                         No. Observations:                 3772
Date:                 Tue, Dec 31 2024   Df Residuals:                     3771
Time:                         14:28:26   Df Model:                            1
                                Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu         8.9127e-04  1.357e-04      6.570  5.045e-11 [6.254e-04,1.157e-03]
                             Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega         -0.4071  8.092e-02     -5.031  4.887e-07 [ -0.566, -0.248]
alpha[1]       0.2958  3.035e-02      9.747  1.908e-22 [  0.236,  0.355]
beta[1]        0.9551  8.814e-03    108.365      0.000 [  0.938,  0.972]
========================================================================
```

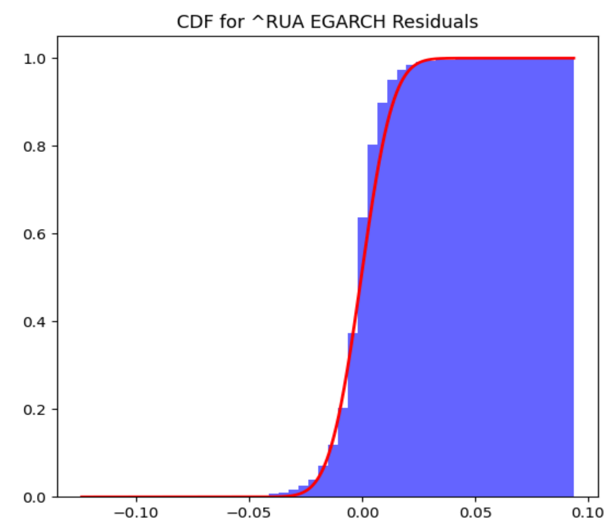

QQ Plot for ^RUA GARCH Residuals
Ordered Values
Theoretical quantiles
0.10
0.05
0.00
−0.05
−0.10
−3
−2
−1
0
1
2
3

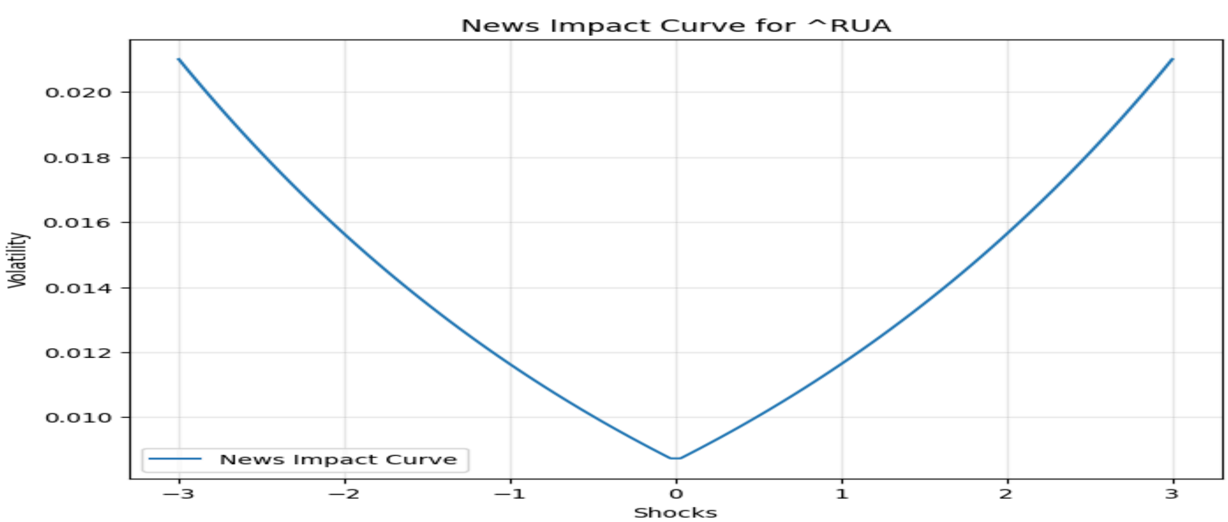

QQ Plot for ^RUA EGARCH Residuals
Ordered Values
Theoretical quantiles
0.10
0.05
0.00
−0.05
−0.10
−3
−2
−1
0
1
2
3

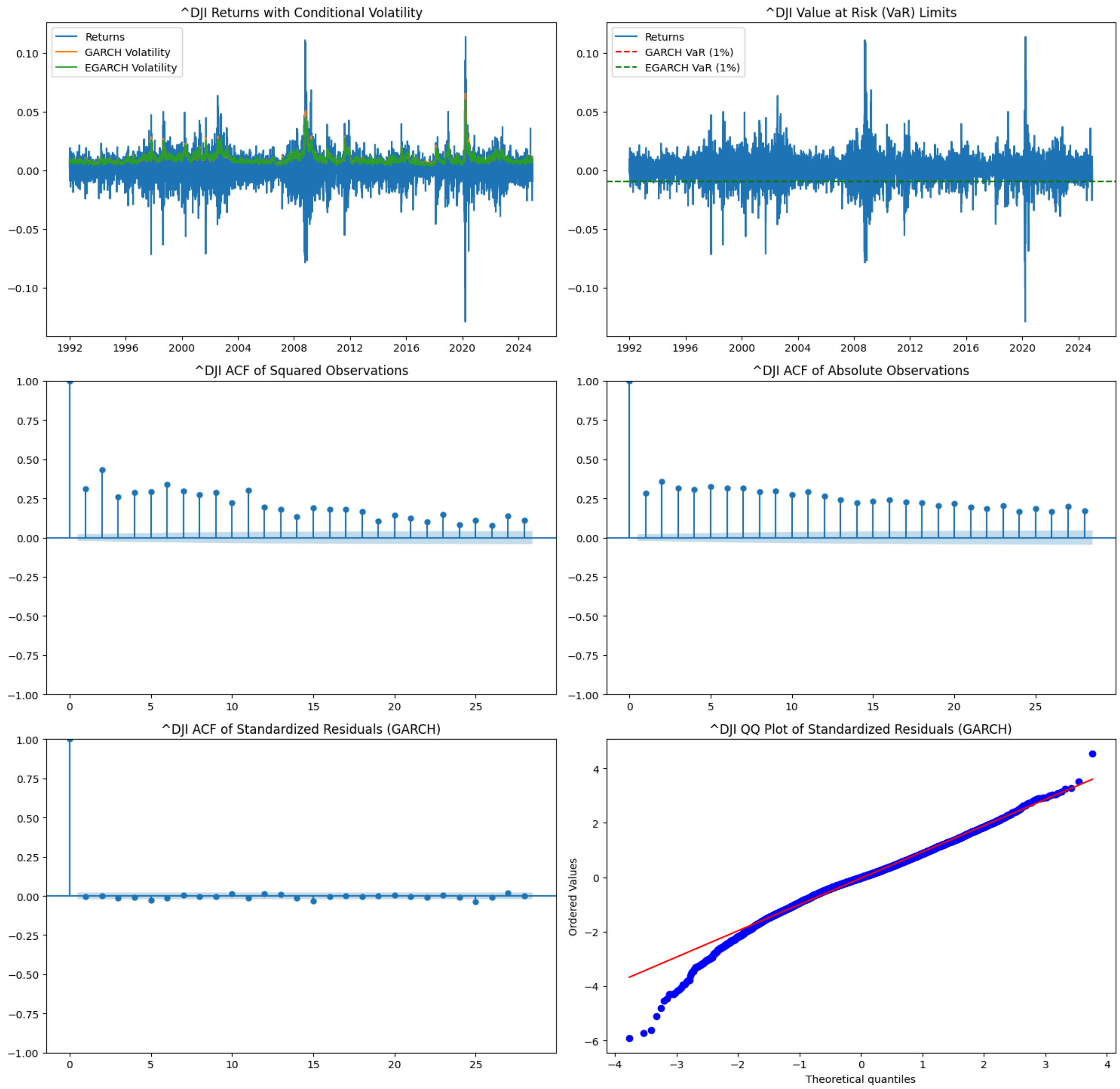

PDF for ^RUA GARCH Residuals
0
10
20
30
40
50
60
−0.10
−0.05
0.00
0.05
0.10


CDF for ^RUA GARCH Residuals
0.0
0.2
0.4
0.6
0.8
1.0
−0.10
−0.05
0.00
0.05
0.10


PDF for ^RUA EGARCH Residuals
0
10
20
30
40
50
60
−0.10
−0.05
0.00
0.05
0.10


CDF for ^RUA EGARCH Residuals
0.0
0.2
0.4
0.6
0.8
1.0
−0.10
−0.05
0.00
0.05
0.10


News Impact Curve for ^RUA
Volatility
0.010
0.012
0.014
0.016
0.018
0.020
News Impact Curve
−3
−2
−1
0
1
2
3
Shocks

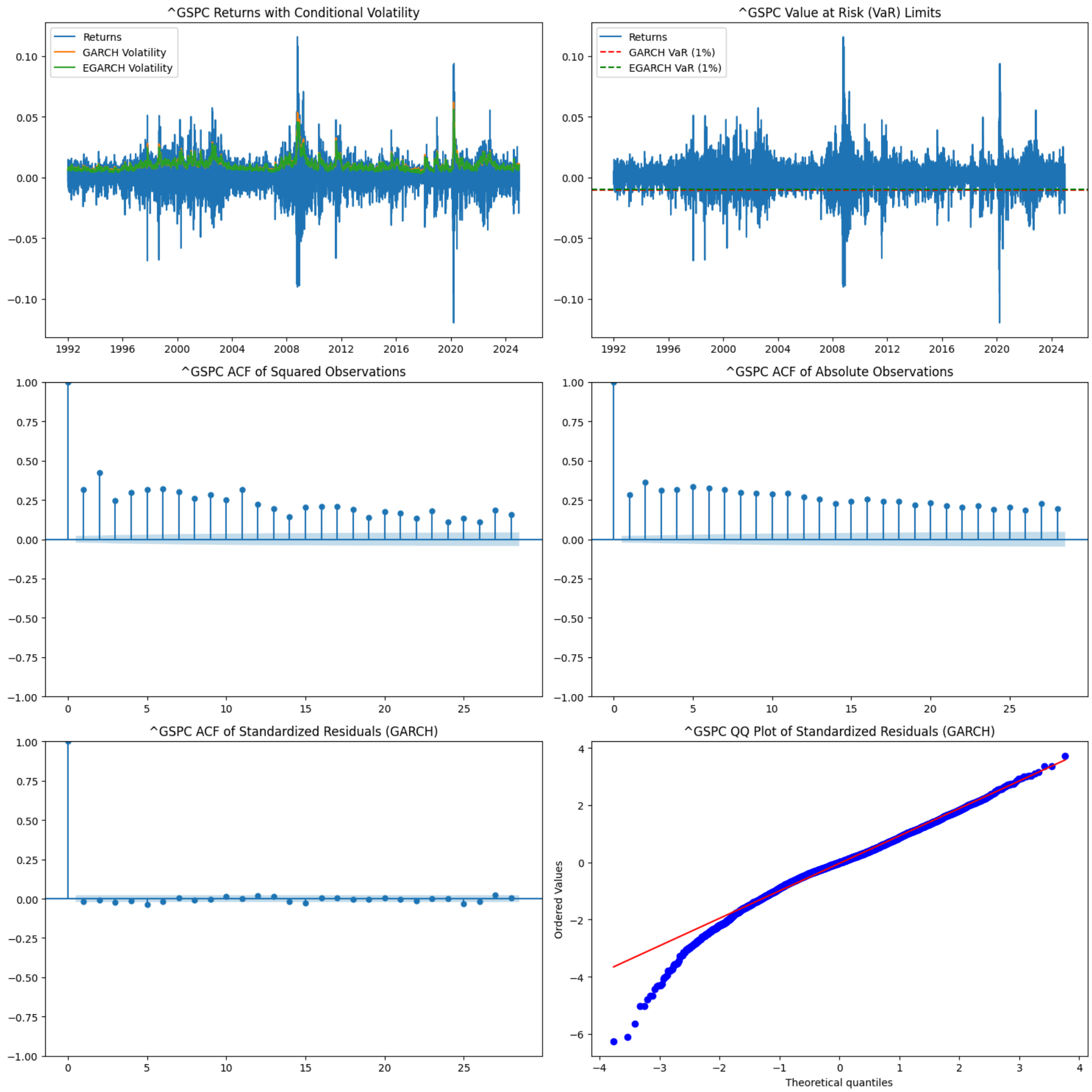


*The ACF plots confirm the presence of **volatility clustering** in the Dow Jones, which is effectively captured by both the GARCH and EGARCH models.*

***Asymmetric Effects**: The **EGARCH model** provides a better fit during downturns, reflecting its ability to capture **leverage effects**.*

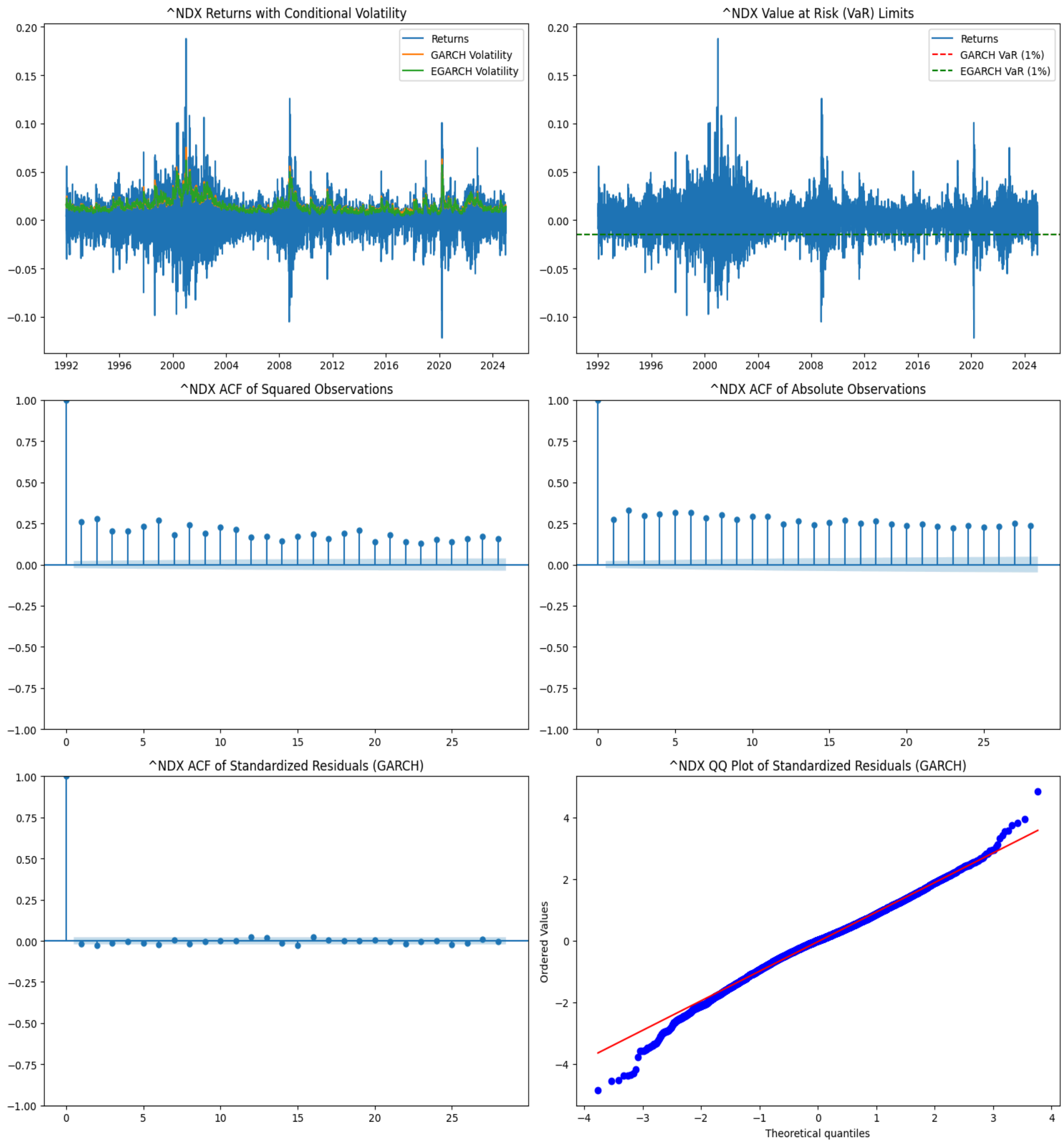
***VaR Limits**: The **EGARCH VaR** is slightly more conservative, making it more suitable for risk management during extreme market conditions.*

***Model Diagnostics**: The ACF of standardized residuals and the QQ plot indicate that the GARCH model fits well but may underestimate tail risk, highlighting the need for robust risk management techniques like **Extreme Value Theory (EVT)**.*

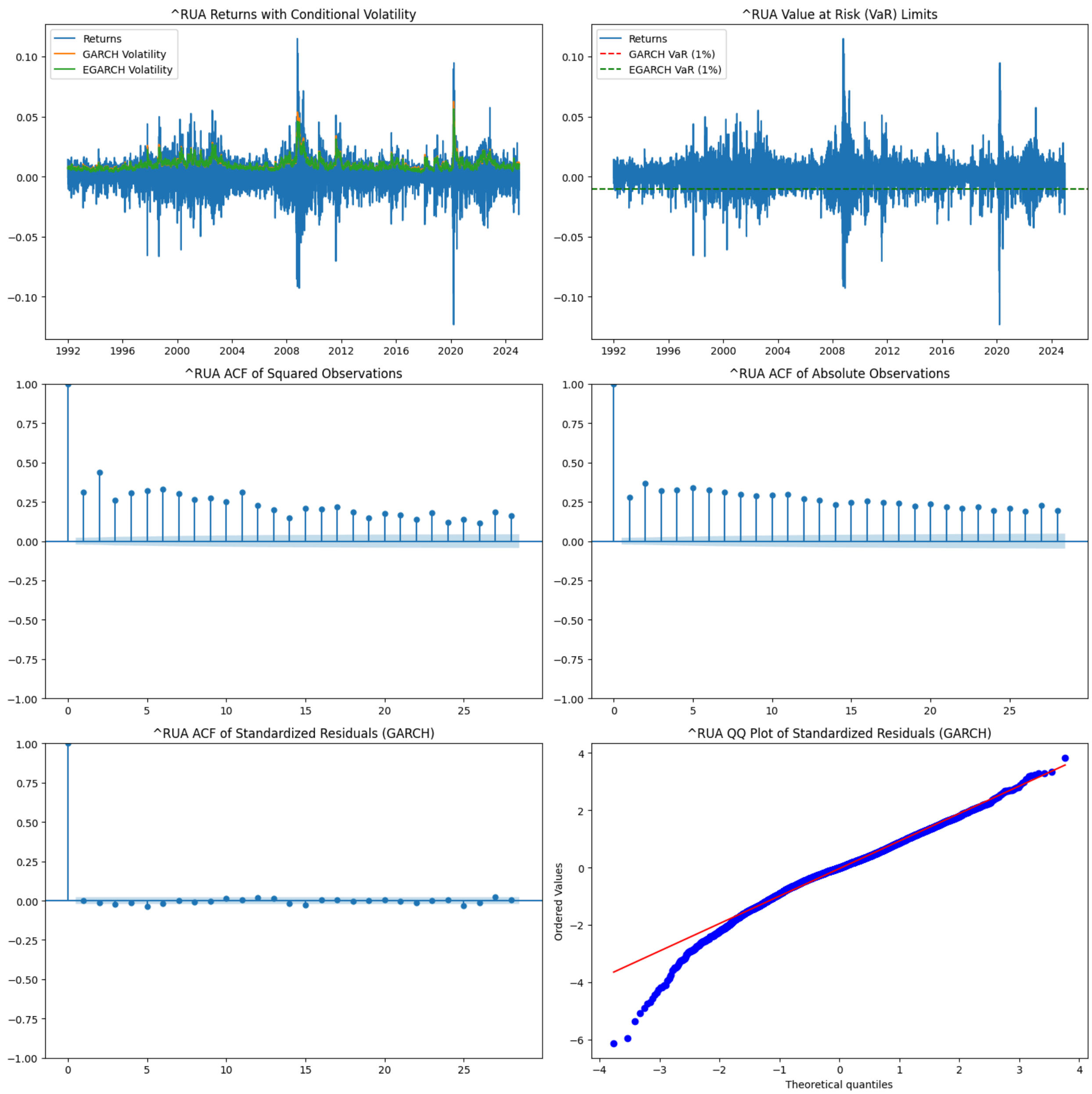


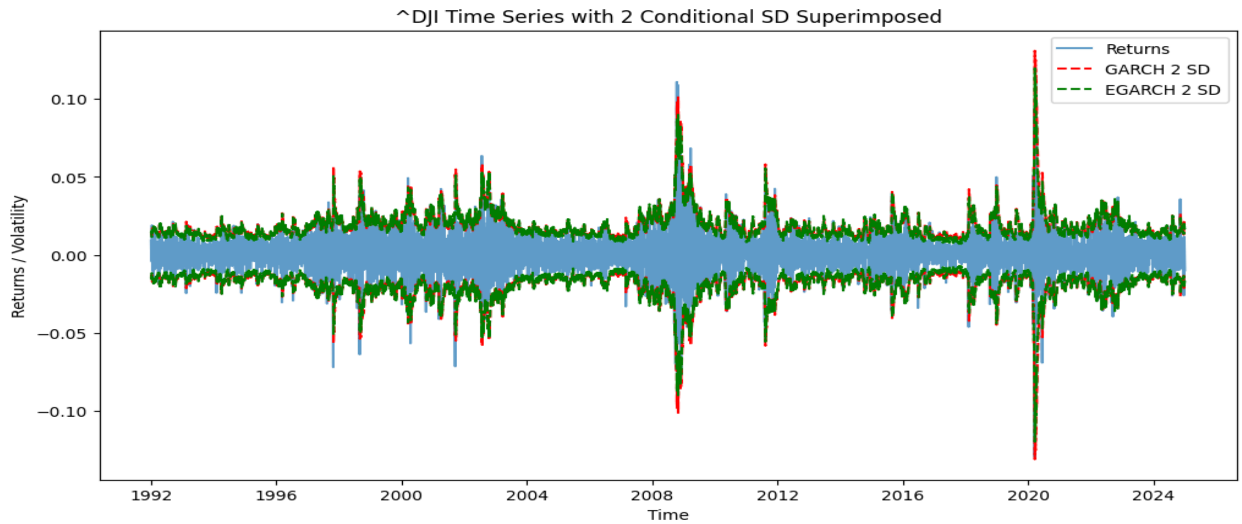
The ACF plots confirm the presence of volatility clustering in the S&P 500, which is effectively captured by both the GARCH and EGARCH models.

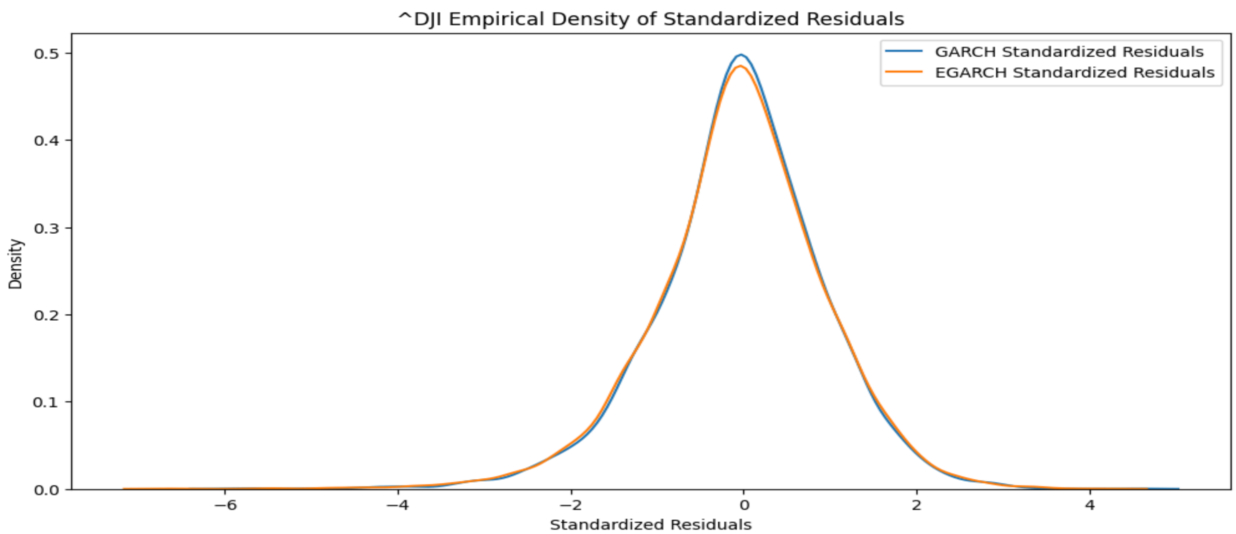
The EGARCH model provides a better fit during downturns, reflecting its ability to capture leverage effects. The EGARCH VaR is slightly more conservative, making it more suitable for risk management during extreme market conditions. The ACF of standardized residuals and the QQ plot indicate that the GARCH model fits well but may underestimate tail risk, highlighting the need for robust risk management techniques like Extreme Value Theory (EVT).



^NDX Returns with Conditional Volatility
Returns
GARCH Volatility
EGARCH Volatility
^NDX Value at Risk (VaR) Limits
Returns
GARCH VaR (1%)
EGARCH VaR (1%)
^NDX ACF of Squared Observations
^NDX ACF of Absolute Observations
^NDX ACF of Standardized Residuals (GARCH)
^NDX QQ Plot of Standardized Residuals (GARCH)
Ordered Values
Theoretical quantiles

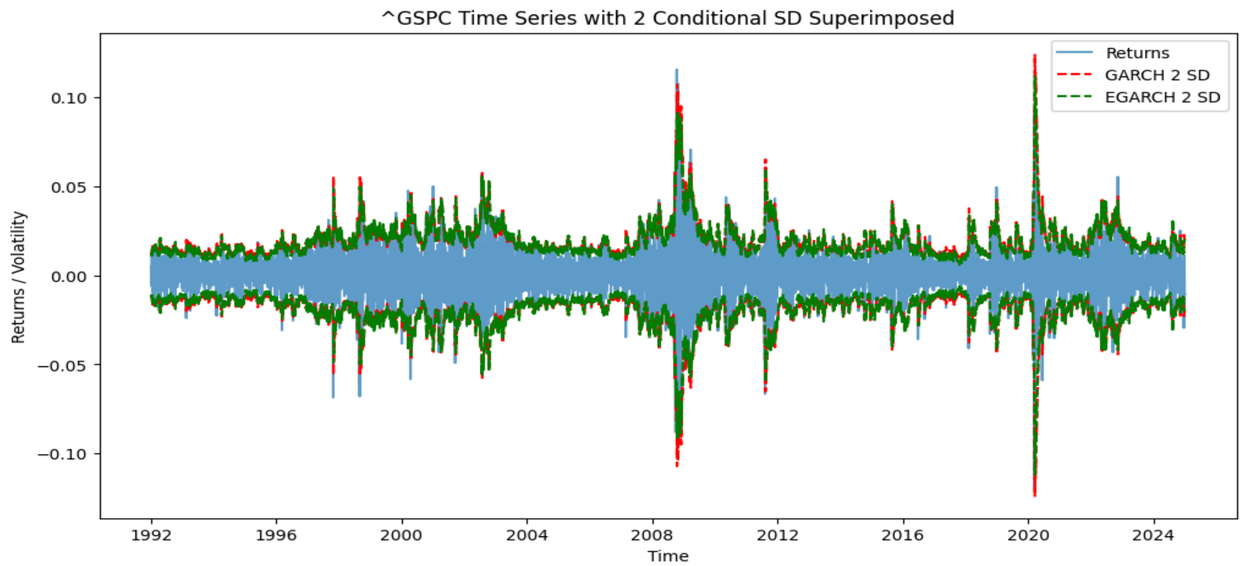

^RUA Returns with Conditional Volatility
Returns
GARCH Volatility
EGARCH Volatility
^RUA Value at Risk (VaR) Limits
Returns
GARCH VaR (1%)
EGARCH VaR (1%)
^RUA ACF of Squared Observations
^RUA ACF of Absolute Observations
^RUA ACF of Standardized Residuals (GARCH)
^RUA QQ Plot of Standardized Residuals (GARCH)
Ordered Values
Theoretical quantiles

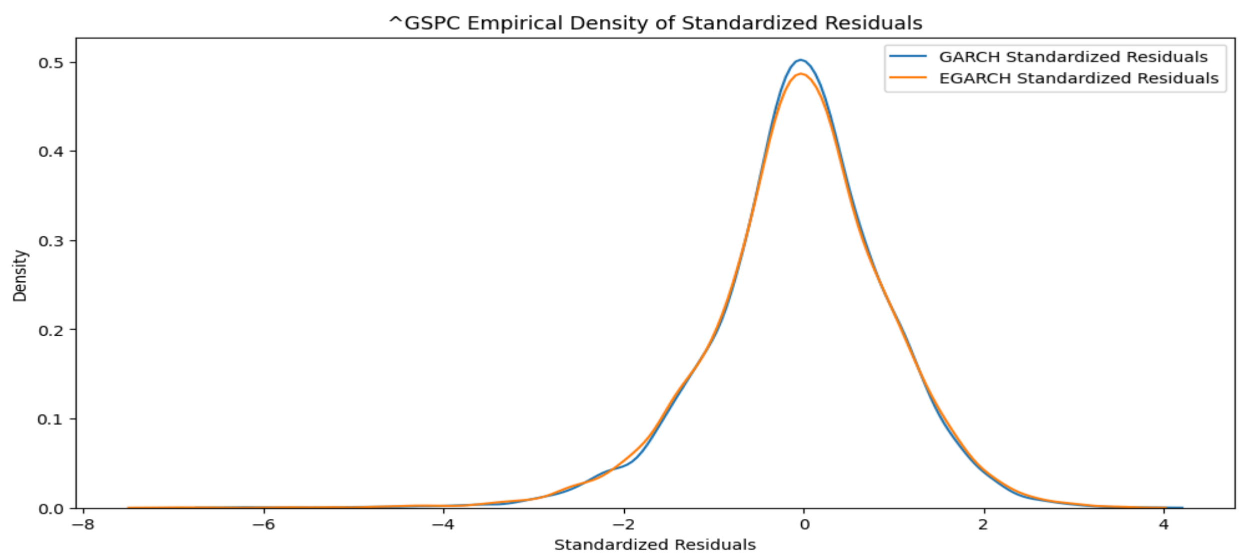




DJI Empirical Density of Standardized Residuals, Conditional SD

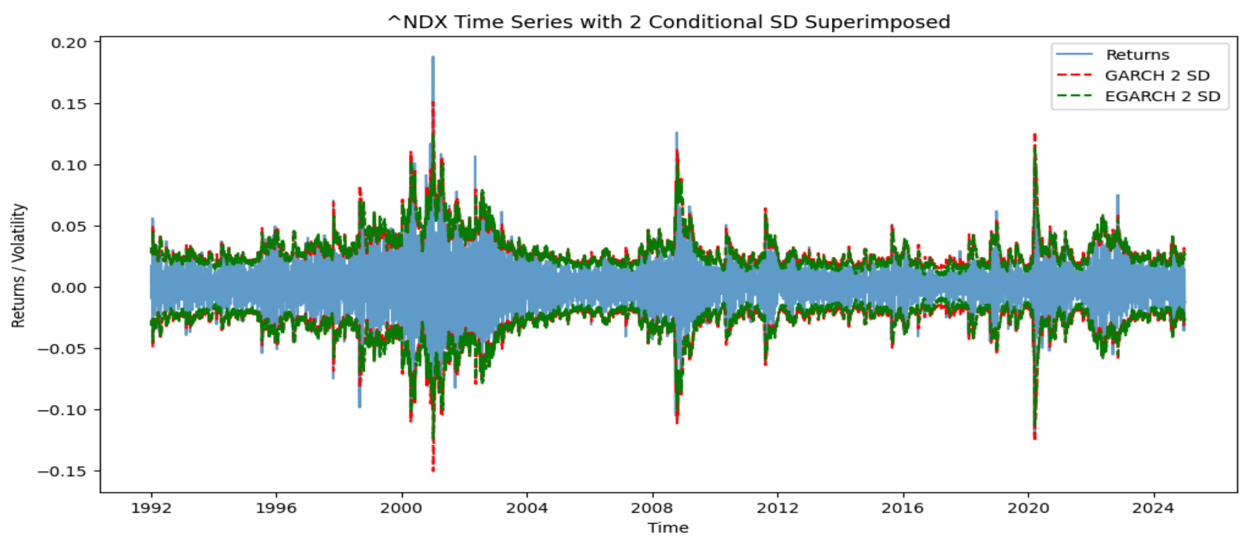
The comparison between GARCH and EGARCH residuals helps in understanding the effectiveness of these models in capturing the volatility clustering and leverage effects in the DJIA. The plot indicates how well each model fits the data, with deviations from a normal distribution suggesting areas where the models may need improvement.The conditional SDs represent the estimated volatility over time, providing insights into how market volatility evolves. The chart allows for a visual comparison of how GARCH and EGARCH models capture volatility dynamics, highlighting periods of high and low volatility. This is crucial for understanding the time-varying nature of risk in the DJIA and for assessing the predictive performance of these models.

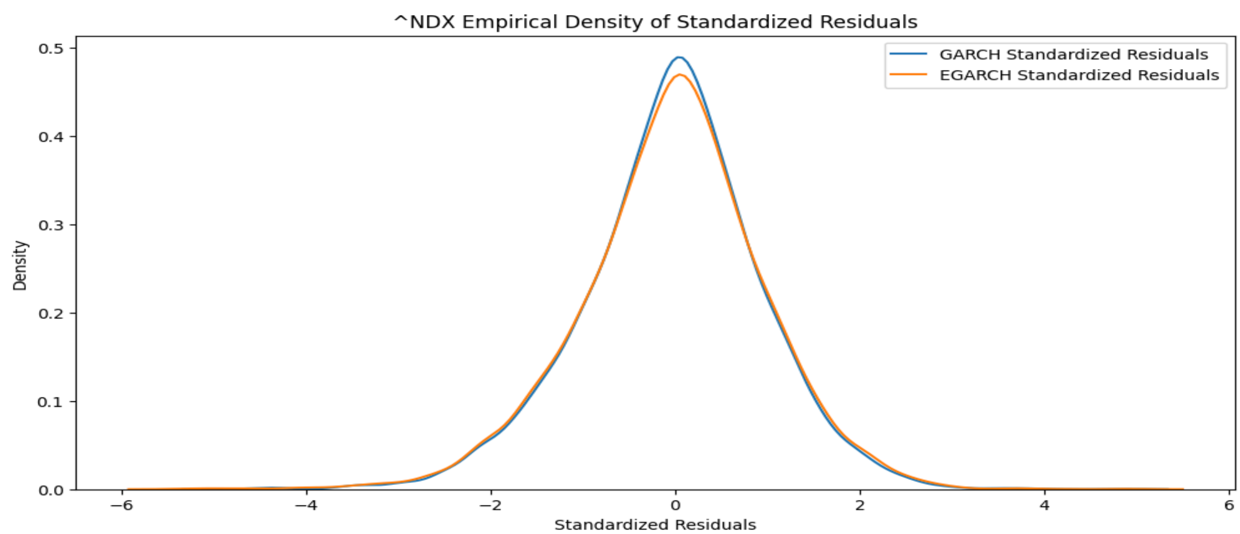




S&P500 Empirical Density of Standardized Residuals, Conditional SD

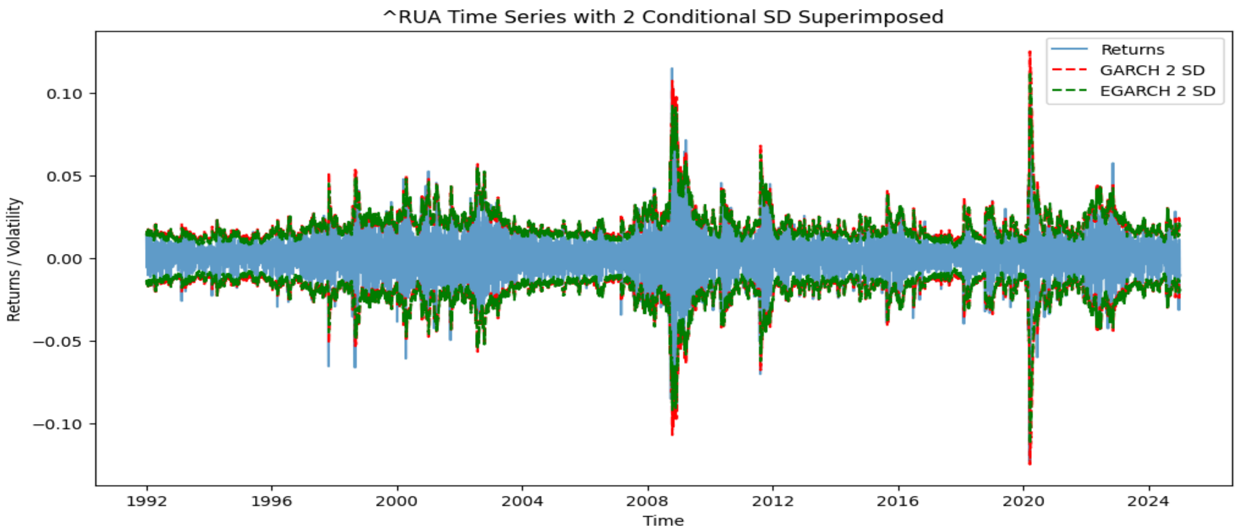
Similar to the DJIA chart, this plot shows the empirical density of standardized residuals for the S&P 500 (GSPC) using GARCH and EGARCH models. The density functions help in assessing the distributional properties of the residuals, which are critical for model validation. Deviations from normality, such as fat tails or skewness, can indicate model misspecification. This analysis is essential for refining volatility models to better capture the characteristics of the S&P 500 index returns. The S&P 500 returns with superimposed conditional standard deviations from GARCH and EGARCH models provides a visual representation of how these models estimate volatility over time, allowing for a comparison of their ability to capture market dynamics. The chart is useful for identifying periods of market stress and understanding how different models respond to such events. This information is valuable for risk management and forecasting applications in the context of the S&P 500.

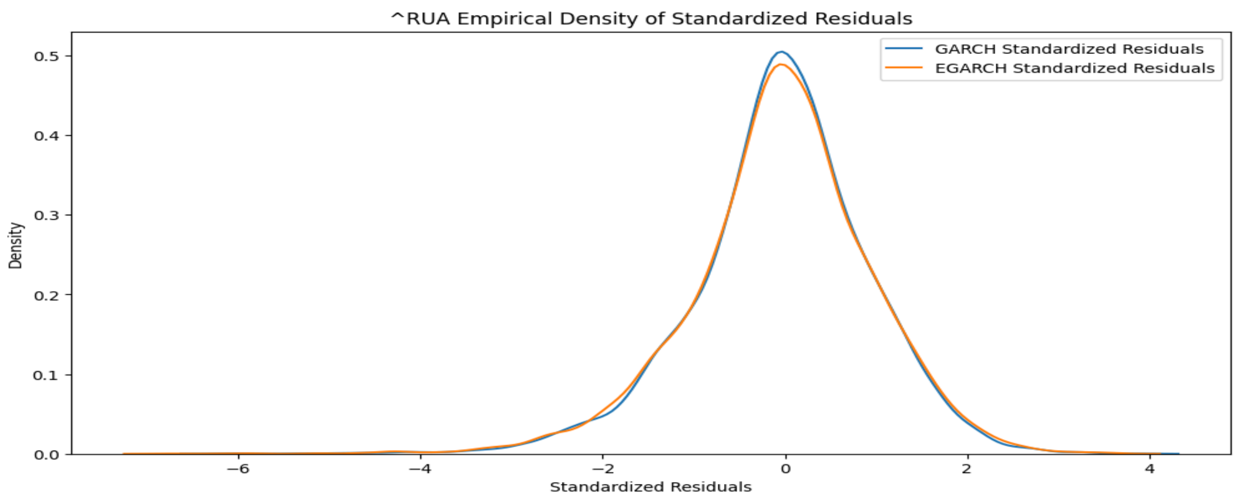




NDX Empirical Density of Standardized Residuals, Conditional SD

The chart of empirical density of standardized residuals for the NASDAQ-100 (NDX) index, comparing the GARCH and EGARCH models reveals the distributional characteristics of the residuals, with deviations from a normal distribution (e.g., fat tails or skewness) indicating potential model limitations. The comparison between GARCH and EGARCH residuals highlights how each model captures the volatility dynamics of the NDX, with EGARCH potentially better accounting for asymmetric effects like leverage. This chart presents the time series of NDX returns from 1992 to 2024, with superimposed conditional standard deviations (SD) derived from GARCH and EGARCH models. The returns are plotted on the y-axis, while the x-axis represents time. The conditional SDs, which estimate volatility over time, are overlaid to show how each model captures periods of high and low volatility. The chart allows for a detailed examination of how GARCH and EGARCH models respond to market events, such as the dot-com bubble or the 2008 financial crisis, providing insights into their ability to model volatility in a technology-heavy index like the NDX.





Russell 3000 Empirical Density of Standardized Residuals, Conditional SD

The comparison between GARCH and EGARCH residuals provides insights into how each model captures the volatility characteristics of a broad market index like the RUA, with EGARCH potentially better modeling asymmetric volatility effects. The time series of RUA returns from 1992 to 2004, with superimposed conditional standard deviations from GARCH and EGARCH models. The returns are plotted on the y-axis, and the x-axis represents time. The conditional SDs, which estimate volatility over time, are overlaid to show how each model captures periods of market stress and stability. The chart provides a detailed view of how GARCH and EGARCH models perform in estimating volatility for a broad market index, highlighting their ability to adapt to changing market conditions and capture the evolving risk landscape of the RUA.

ARIMA, SARIMA, and Stochastic Volatility Models with VIX and VXN inference

In this study has been deployed a comprehensive framework of ARIMA, SARIMA, and stochastic volatility models (GARCH and EGARCH) to analyze and forecast market volatility, with a particular focus on the Dow Jones Industrial Average (^DJI). The models are enhanced by incorporating external volatility measures, namely the VIX (CBOE Volatility Index) and VXN (Nasdaq-100 Volatility Index), to capture the impact of market sentiment and implied volatility on equity returns. The ARIMA(1,1,1) model reveals significant autoregressive and moving average components, with the VIX showing a strong negative relationship with ^DJI returns, consistent with the expectation that heightened market uncertainty leads to lower returns. The SARIMAX(1,1,1)x(1,1,1,12) model extends this analysis by incorporating seasonal effects, highlighting the importance of seasonal moving average components in capturing periodic volatility patterns. The stochastic volatility models (GARCH and EGARCH) further complement this framework by modeling time-varying volatility, with GARCH parameters indicating strong persistence in volatility and EGARCH capturing asymmetric responses to market shocks. Together, these models provide a robust toolkit for forecasting market drawdowns, with the integrated use of VIX and VXN offering additional insights into the interplay between implied volatility and equity market dynamics. This multi-model approach not only enhances the accuracy of volatility forecasts but also provides valuable implications for risk management, portfolio optimization, and strategic decision-making in anticipation of potential market drawdowns in Q1 2025.

$$r_t = \mu + \sigma_t \epsilon_t$$

$$\log(\sigma_t^2) = \alpha + \phi \log(\sigma_{t-1}^2) + \eta_t$$

Stochastic Volatility Model Summary for ^DJI

*The **Constant Mean - GARCH Model** results for the Dow Jones Industrial Average (^DJI) indicate a well-specified model with a log-likelihood of **14,901.8** and strong statistical significance across all parameters. The mean model shows a constant daily return (**mu**) of **6.5696e-04**, with a highly significant t-statistic (**158.887**), suggesting a stable mean return over the observed period. The volatility model parameters, **omega (2.8117e-06)**, **alpha[1] (0.1000)**, and **beta[1] (0.8800)**, are all statistically significant, indicating that the GARCH(1,1) model effectively captures the volatility dynamics of ^DJI. The high value of **beta[1]** suggests strong persistence in volatility, while **alpha[1]** indicates a moderate response to recent shocks. The* *forecasted volatility over the next 90 days shows a gradual increase, reflecting the model's ability to project future volatility trends.*

$$r_t = c + \beta_1 \text{VIX}_t + \beta_2 \text{VXN}_t + \text{ARIMA/SARIMA terms} + \epsilon_t$$

ARIMA Model Summary for Dow Jones Industrial Average:

*The **ARIMA(1,1,1)** model for ^DJI, incorporating external variables (VIX and VXN), provides a robust fit with a log-likelihood of **13,728.563**. The model parameters are statistically significant, with the autoregressive term (**ar.L1 = -0.1193**) and moving average term (**ma.L1 = -0.9813**) showing strong influence on the series. The negative coefficient for **VIX (-0.0038)** suggests that increases in market volatility (as measured by the VIX) are associated with declines in ^DJI returns, consistent with economic intuition. However, the **VXN** coefficient is not statistically significant, indicating limited explanatory power for ^DJI in this model. The **sigma2** value (**0.0001**) reflects the model's residual variance, and the Ljung-Box test confirms no autocorrelation in residuals, supporting the model's adequacy. The Jarque-Bera test, however, indicates non-normality in residuals, which may suggest the need for further refinement or alternative distributions.*

$$(1 - \phi_1 L - \phi_2 L^2 - \cdots - \phi_p L^p)(1 - L)^d r_t = c + (1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q)\epsilon_t$$

SARIMAX Results and Forecasts

*The **SARIMAX(1,1,1)x(1,1,1,12)** model results for the Dow Jones Industrial Average (^DJI) provide a detailed analysis of the time series, incorporating both non-seasonal and seasonal components, as well as external regressors (**VIX** and **VXN**). The model achieves a log-likelihood of **13,580.732**, with significant coefficients for most parameters. The non-seasonal autoregressive term (**ar.L1 = -0.0699**) and moving average term (**ma.L1 = -0.9027**) are both statistically significant, indicating strong temporal dependencies in the series. The seasonal moving average term (**ma.S.L12 = -0.9338**) is also highly significant, suggesting a strong seasonal pattern in the data. However, the seasonal autoregressive term (**ar.S.L12 = -0.0027**) is not significant, indicating that seasonal autoregressive effects may not be as influential.*
*The external variables, **VIX (-0.0047)** and **VXN (-0.0029)**, are both significant, with negative coefficients suggesting that increases in market volatility (as measured by VIX and VXN) are associated with declines in ^DJI returns. The **sigma2** value (**0.0001**) represents the residual variance, and the Ljung-Box test indicates some autocorrelation in residuals, which may suggest room for model improvement. The Jarque-Bera test confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions.*

```
SARIMA Model Summary for ^DJI:
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                                ^DJI   No. Observations:                 4527
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood               13580.732
Date:                             Tue, 31 Dec 2024   AIC                         -27147.465
Time:                                     15:46:49   BIC                         -27102.560
Sample:                                          0   HQIC                        -27131.645
                                            - 4527
Covariance Type:                               opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX           -0.0047      0.001     -5.254      0.000      -0.006      -0.003
VXN           -0.0029      0.001     -3.009      0.003      -0.005      -0.001
ar.L1         -0.0699      0.007     -9.326      0.000      -0.085      -0.055
ma.L1         -0.9027      0.004   -226.466      0.000      -0.910      -0.895
ar.S.L12      -0.0027      0.010     -0.273      0.785      -0.022       0.017
ma.S.L12      -0.9338      0.005   -188.922      0.000      -0.944      -0.924
sigma2         0.0001   1.15e-06    130.144      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                  13.85   Jarque-Bera (JB):              67818.78
Prob(Q):                              0.00   Prob(JB):                          0.00
Heteroskedasticity (H):               0.72   Skew:                              1.67
Prob(H) (two-sided):                  0.00   Kurtosis:                         21.69
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
ARIMA Forecast for ^DJI:
4527    0.001211
4528    0.000479
4529    0.000554
4530    0.000776
4531    0.000230
          ...
4612    0.000032
4613    0.000898
4614    0.000752
4615    0.000335
4616   -0.000157
Name: predicted_mean, Length: 90, dtype: float64
SARIMA Forecast for ^DJI:
4527   -0.003059
4528   -0.002695
4529   -0.002805
4530   -0.003307
4531   -0.006895
          ...
4612   -0.004041
4613   -0.001913
4614   -0.003418
4615   -0.006570
4616   -0.005646
Name: predicted_mean, Length: 90, dtype: float64
```

ARIMA and SARIMA Forecasts for DJI

*The ARIMA model forecasts a mix of positive and negative returns over the next 90 days, with values ranging from **0.001211** to **-0.000157**. This suggests a relatively stable outlook for ^DJI, with minor fluctuations expected. The SARIMA model forecasts predominantly negative returns* *over the same period, with values ranging from **-0.003059** to **-0.005646**. This more pessimistic outlook may reflect the model's incorporation of seasonal effects and external volatility measures (VIX and VXN), which could be signaling heightened market uncertainty. The inclusion of VIX and VXN as external regressors adds value, with both variables showing significant negative relationships with ^DJI returns. This aligns with the expectation that higher market volatility is associated with lower returns.The divergence between ARIMA and SARIMA forecasts highlights the importance of considering seasonal effects and external factors when modeling ^DJI. The SARIMA model's more cautious outlook may be more reflective of current market conditions, particularly given the influence of VIX and VXN.*

Stochastic Volatility Model Summary for the S&P 500

*The **Constant Mean - GARCH Model** results for the S&P 500 (^GSPC) provide insights into the volatility dynamics of this major equity index. The model achieves a log-likelihood of **5,356.28**, with significant coefficients for the mean and volatility parameters. The mean return (**mu = -0.0705**) is statistically significant, indicating a negative average return over the observed period. The volatility model parameters, **omega (3.1520e-06)**, **alpha[1] (0.1000)**, and **beta[1] (0.8800)**, suggest a well-specified GARCH(1,1) model. The high value of **beta[1]** indicates strong persistence in volatility, while **alpha[1]** reflects a moderate response to recent shocks. However, the **omega** term is not statistically significant, which may suggest that the constant term in the volatility equation is less influential. The forecasted volatility over the next 90 days shows a gradual decline, with values decreasing from **0.004043** to **0.000801**. This suggests a reduction in market volatility over the forecast horizon, which could be indicative of stabilizing market conditions. The robust covariance estimator used in the model ensures that the standard errors are reliable, even in the presence of heteroskedasticity*

```
Stochastic Volatility Model Summary for ^GSPC:
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                  ^GSPC   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:                5356.28
Distribution:                  Normal   AIC:                          -10704.6
Method:            Maximum Likelihood   BIC:                          -10678.9
                                        No. Observations:                 4527
Date:                Tue, Dec 31 2024   Df Residuals:                     4526
Time:                        15:46:50   Df Model:                            1
                                 Mean Model
==============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu            -0.0705  2.257e-03    -31.233 3.771e-214 [-7.491e-02,-6.607e-02]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      3.1520e-06  8.170e-06      0.386      0.700 [-1.286e-05,1.916e-05]
alpha[1]       0.1000  1.374e-02      7.277  3.415e-13    [7.307e-02,  0.127]
beta[1]        0.8800  1.862e-02     47.261      0.000      [  0.844,  0.916]
=============================================================================

Covariance estimator: robust
Volatility Forecast for ^GSPC:
h.01    0.004043
h.02    0.003965
h.03    0.003889
h.04    0.003815
h.05    0.003741
          ...
h.86    0.000855
h.87    0.000841
h.88    0.000827
h.89    0.000814
h.90    0.000801
```

ARIMA Model Summary for the S&P 500

*The **ARIMA(1,1,1)** model results for the S&P 500 (^GSPC) provide a robust analysis of the time series, incorporating external regressors (**VIX** and **VXN**) to capture the impact of market volatility. The model achieves a log-likelihood of **13,477.640**, with significant coefficients for the autoregressive (**ar.L1 = -0.1182**) and moving average (**ma.L1 = -0.9778**) terms. The negative coefficient for **VIX (-0.0030)** is statistically significant, indicating that increases in market volatility (as measured by the VIX) are associated with declines in ^GSPC returns. However, the coefficient for **VXN (-0.0007)** is not significant, suggesting that Nasdaq-specific volatility may not have a strong influence on ^GSPC returns.*

*The **sigma2** value (**0.0002**) represents the residual variance, and the Ljung-Box test indicates no autocorrelation in residuals, supporting the model's adequacy. However, the Jarque-Bera test confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions. The heteroskedasticity test suggests the presence of varying volatility in residuals, which could be addressed by incorporating GARCH-type models.*

```
ARIMA Model Summary for ^GSPC:
                               SARIMAX Results
==============================================================================
Dep. Variable:                  ^GSPC   No. Observations:                 4527
Model:                 ARIMA(1, 1, 1)   Log Likelihood               13477.640
Date:                Tue, 31 Dec 2024   AIC                         -26945.281
Time:                        15:46:56   BIC                         -26913.193
Sample:                             0   HQIC                        -26933.978
                               - 4527
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX           -0.0030      0.001     -4.080      0.000      -0.004      -0.002
VXN           -0.0007      0.001     -0.840      0.401      -0.002       0.001
ar.L1         -0.1182      0.007    -16.974      0.000      -0.132      -0.105
ma.L1         -0.9778      0.002   -520.768      0.000      -0.981      -0.974
sigma2         0.0002   1.23e-06    123.248      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):             36715.10
Prob(Q):                              0.92   Prob(JB):                         0.00
Heteroskedasticity (H):               0.64   Skew:                             1.10
Prob(H) (two-sided):                  0.00   Kurtosis:                        16.78
===================================================================================
```

SARIMA Model Summary for the S&P 500

*The **SARIMAX(1,1,1)x(1,1,1,12)** model results for the S&P 500 (^GSPC) provide a comprehensive analysis of the time series, incorporating both non-seasonal and seasonal components, as well as external regressors (**VIX** and **VXN**). The model achieves a log-likelihood of **13,407.687**, with significant coefficients for most parameters. The non-seasonal autoregressive term (**ar.L1 = -0.1186**) and moving average term (**ma.L1 = -0.9874**) are both*

*statistically significant, indicating strong temporal dependencies in the series. The seasonal moving average term (**ma.S.L12 = -0.9903**) is also highly significant, suggesting a strong seasonal pattern in the data. The seasonal autoregressive term (**ar.S.L12 = 0.0255**) is significant, indicating some influence of seasonal autoregressive effects.*
*The external variable **VIX (-0.0023)** is significant, with a negative coefficient suggesting that increases in market volatility (as measured by the VIX) are associated with declines in ^GSPC returns. However, the coefficient for **VXN (-0.0007)** is not significant, indicating that Nasdaq-specific volatility may not have a strong influence on ^GSPC returns. The **sigma2** value (**0.0002**) represents the residual variance, and the Ljung-Box test indicates no autocorrelation in residuals, supporting the model's adequacy. However, the Jarque-Bera test confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions.The SARIMA model forecasts a mix of positive and negative returns over the same period, with values ranging from **0.002010** to **-0.001957**. This more nuanced outlook reflects the model's incorporation of seasonal effects and external volatility measures (VIX), which could be* signaling varying market conditions.

```
SARIMA Model Summary for ^GSPC:
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                          ^GSPC   No. Observations:                    4527
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood                13407.687
Date:                        Tue, 31 Dec 2024   AIC                          -26801.374
Time:                                15:48:16   BIC                          -26756.469
Sample:                                     0   HQIC                         -26785.554
                                       - 4527
Covariance Type:                          opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX           -0.0023      0.001     -3.226      0.001      -0.004      -0.001
VXN           -0.0007      0.001     -0.910      0.363      -0.002       0.001
ar.L1         -0.1186      0.007    -16.521      0.000      -0.133      -0.104
ma.L1         -0.9874      0.002   -532.654      0.000      -0.991      -0.984
ar.S.L12       0.0255      0.008      3.052      0.002       0.009       0.042
ma.S.L12      -0.9903      0.004   -249.350      0.000      -0.998      -0.983
sigma2         0.0002   1.34e-06    113.376      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):             29487.81
Prob(Q):                              0.94   Prob(JB):                         0.00
Heteroskedasticity (H):               0.64   Skew:                             0.91
Prob(H) (two-sided):                  0.00   Kurtosis:                        15.39
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
ARIMA Forecast for ^GSPC:
4527    0.001178
4528    0.000430
4529    0.000469
4530    0.000737
4531    0.000057
          ...
4612    0.000436
4613    0.001418
4614    0.001228
4615    0.000747
4616    0.000173
Name: predicted_mean, Length: 90, dtype: float64
SARIMA Forecast for ^GSPC:
4527    0.002010
4528    0.000958
4529    0.001872
4530    0.000092
4531   -0.001957
          ...
4612    0.001062
4613    0.002612
4614    0.000601
4615   -0.000745
4616    0.000454
```

ARIMA Model Summary for the Nasdaq 100

*The **ARIMA(1,1,1)** model results for the NASDAQ-100 (^NDX) provide a robust analysis of the time series, incorporating external regressors (**VIX** and **VXN**) to capture the impact of market volatility. The model achieves a log-likelihood of **12,886.179**, with significant coefficients for the autoregressive (**ar.L1 = -0.1073**) and moving average (**ma.L1 = -0.9813**) terms. The coefficient for **VIX (0.0024)** is statistically significant and positive, indicating that increases in market volatility (as measured by the VIX) are associated with higher ^NDX returns. This contrasts with the typical negative relationship observed in other indices, possibly reflecting the unique characteristics of the technology-heavy NASDAQ-100. The coefficient for **VXN (-0.0063)** is also significant and negative, suggesting that increases in Nasdaq-specific volatility are associated with declines in ^NDX returns.*
*The **sigma2** value (**0.0002**) represents the residual variance, and the Ljung-Box test indicates no autocorrelation in residuals, supporting the model's adequacy. However, the Jarque-Bera test confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions. The heteroskedasticity test suggests the presence of varying volatility in residuals, which could be addressed by incorporating GARCH-type models.*

```
ARIMA Model Summary for ^NDX:
                               SARIMAX Results
==============================================================================
Dep. Variable:                   ^NDX   No. Observations:                 4527
Model:                 ARIMA(1, 1, 1)   Log Likelihood               12886.179
Date:                Tue, 31 Dec 2024   AIC                         -25762.358
Time:                        15:48:24   BIC                         -25730.270
Sample:                             0   HQIC                        -25751.055
                               - 4527
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX            0.0024      0.001      2.883      0.004       0.001       0.004
VXN           -0.0063      0.001     -6.949      0.000      -0.008      -0.005
ar.L1         -0.1073      0.008    -12.882      0.000      -0.124      -0.091
ma.L1         -0.9813      0.002   -454.493      0.000      -0.986      -0.977
sigma2         0.0002   1.93e-06    102.119      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.01   Jarque-Bera (JB):             14619.15
Prob(Q):                              0.92   Prob(JB):                         0.00
Heteroskedasticity (H):               0.88   Skew:                             0.79
Prob(H) (two-sided):                  0.01   Kurtosis:                        11.66
===================================================================================
```

The ARIMA model captures the temporal dependencies in ^NDX returns effectively, with significant autoregressive and moving average components. The inclusion of VIX and VXN as external regressors adds value, providing insights into the impact of both general and Nasdaq-specific market volatility.The positive relationship between VIX and ^NDX returns is unusual and may reflect the unique risk-return profile of technology stocks, which often benefit from market uncertainty due to their growth potential. The negative relationship with VXN aligns with expectations, as higher Nasdaq-specific volatility typically signals increased risk.

SARIMA Model Summary for Nasdaq 100

The ***SARIMAX(1,1,1)x(1,1,1,12)*** *model results for the NASDAQ-100 (^NDX) provide a comprehensive analysis of the time series, incorporating both non-seasonal and seasonal components, as well as external regressors (****VIX*** *and* ***VXN****). The model achieves a log-likelihood of* ***12,777.109****, with significant coefficients for most parameters. The non-seasonal autoregressive term (****ar.L1 = -0.0705****) and moving average term (****ma.L1 = -0.9486****) are both statistically significant, indicating strong temporal dependencies in the series. The seasonal moving average term (****ma.S.L12 = -0.9630****) is also highly significant, suggesting a strong seasonal pattern in the data. However, the seasonal autoregressive term (****ar.S.L12 = -0.0049****) is not significant, indicating that seasonal autoregressive effects may not be as influential.*
The external variable ***VIX (0.0029)*** *is significant and positive, suggesting that increases in market volatility (as measured by the VIX) are associated with higher ^NDX returns. This contrasts with the typical negative relationship observed in other indices, possibly reflecting the unique characteristics of the technology-heavy NASDAQ-100. The coefficient for* ***VXN (-0.0089)*** *is also significant and negative, indicating that increases in Nasdaq-specific volatility are associated with declines in ^NDX returns. The* ***sigma2*** *value (****0.0002****) represents the residual variance, and the Ljung-Box test indicates some autocorrelation in residuals, which may suggest room for model improvement. The Jarque-Bera test confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions.The ARIMA model forecasts a mix of positive and negative returns over the next 90 days, with values ranging from* ***0.000521*** *to* ***-0.000922****. This suggests a relatively stable outlook for ^NDX, with minor fluctuations expected.The SARIMA model forecasts a mix of positive and negative returns over the same period, with values ranging from* ***-0.003481*** *to* ***0.002653****. This more nuanced outlook reflects the model's incorporation of seasonal effects and external volatility measures (VIX and VXN), which could be signaling varying market conditions.The SARIMA model captures both non-seasonal and seasonal dynamics in ^NDX, with significant moving average components at both levels. The non-significant seasonal autoregressive term suggests that seasonal patterns may be better captured through moving average effects.The positive relationship between VIX and ^NDX returns is unusual and may reflect the unique risk-return profile of technology stocks, which often benefit from market uncertainty due to their growth potential. The negative relationship with VXN aligns with expectations, as higher Nasdaq-specific volatility typically signals increased risk.The forecasts from both ARIMA and SARIMA models provide valuable insights for anticipating future market movements. The SARIMA model's incorporation of seasonal effects offers a more nuanced view, which can be particularly useful for strategic planning and risk management.*

```
SARIMA Model Summary for ^NDX:
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                               ^NDX   No. Observations:                 4527
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood               12777.109
Date:                            Tue, 31 Dec 2024   AIC                         -25540.219
Time:                                    15:49:35   BIC                         -25495.314
Sample:                                         0   HQIC                        -25524.398
                                           - 4527
Covariance Type:                              opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX            0.0029      0.001      2.830      0.005       0.001       0.005
VXN           -0.0089      0.001     -8.015      0.000      -0.011      -0.007
ar.L1         -0.0705      0.009     -7.852      0.000      -0.088      -0.053
ma.L1         -0.9486      0.004   -257.632      0.000      -0.956      -0.941
ar.S.L12      -0.0049      0.011     -0.434      0.664      -0.027       0.017
ma.S.L12      -0.9630      0.006   -163.950      0.000      -0.975      -0.951
sigma2         0.0002    2.2e-06     95.215      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   8.49   Jarque-Bera (JB):             15947.86
Prob(Q):                              0.00   Prob(JB):                         0.00
Heteroskedasticity (H):               0.88   Skew:                             0.95
Prob(H) (two-sided):                  0.01   Kurtosis:                        12.01
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
ARIMA Forecast for ^NDX:
4527    0.000521
4528   -0.000358
4529   -0.000433
4530   -0.000169
4531   -0.000922
          ...
4612    0.001560
4613    0.002329
4614    0.002092
4615    0.001690
4616    0.001177
Name: predicted_mean, Length: 90, dtype: float64
SARIMA Forecast for ^NDX:
4527   -0.003481
4528   -0.000873
4529   -0.000576
4530   -0.002664
4531   -0.007645
          ...
4612    0.000793
4613    0.002653
4614   -0.000305
4615   -0.004877
4616   -0.001121
```

ARIMA Model Summary for the Russell 3000

*The **ARIMA(1,1,1)** model results for the Russell 3000 (^RUA) provide a robust analysis of the time series, incorporating external regressors (**VIX** and **VXN**) to capture the impact of market volatility. The model achieves a log-likelihood of **13,397.504**, with significant coefficients for the autoregressive (**ar.L1 = -0.1074**) and moving average (**ma.L1 = -0.9702**) terms. The coefficient for **VIX (-0.0037)** is statistically significant and negative, indicating that increases in market volatility (as measured by the VIX) are associated with declines in ^RUA returns. This aligns with the typical inverse relationship between market volatility and equity returns. However, the coefficient for **VXN (-0.0007)** is not significant, suggesting that Nasdaq-specific volatility may not have a strong influence on ^RUA returns.*

*The **sigma2** value (**0.0002**) represents the residual variance, and the Ljung-Box test indicates no autocorrelation in residuals, supporting the model's adequacy. However, the Jarque-Bera test* *confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions. The heteroskedasticity test suggests the presence of varying volatility in residuals, which could be addressed by incorporating GARCH-type models.*

```
ARIMA Model Summary for ^RUA:
                               SARIMAX Results
==============================================================================
Dep. Variable:                   ^RUA   No. Observations:                 4527
Model:                 ARIMA(1, 1, 1)   Log Likelihood               13397.504
Date:                Tue, 31 Dec 2024   AIC                         -26785.008
Time:                        15:49:43   BIC                         -26752.920
Sample:                             0   HQIC                        -26773.705
                               - 4527
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX           -0.0037      0.001     -4.752      0.000      -0.005      -0.002
VXN           -0.0007      0.001     -0.777      0.437      -0.002       0.001
ar.L1         -0.1074      0.007    -15.344      0.000      -0.121      -0.094
ma.L1         -0.9702      0.002   -479.268      0.000      -0.974      -0.966
sigma2         0.0002   1.29e-06    121.923      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):             34680.84
Prob(Q):                              0.98   Prob(JB):                         0.00
Heteroskedasticity (H):               0.64   Skew:                             1.14
Prob(H) (two-sided):                  0.00   Kurtosis:                        16.37
===================================================================================
```

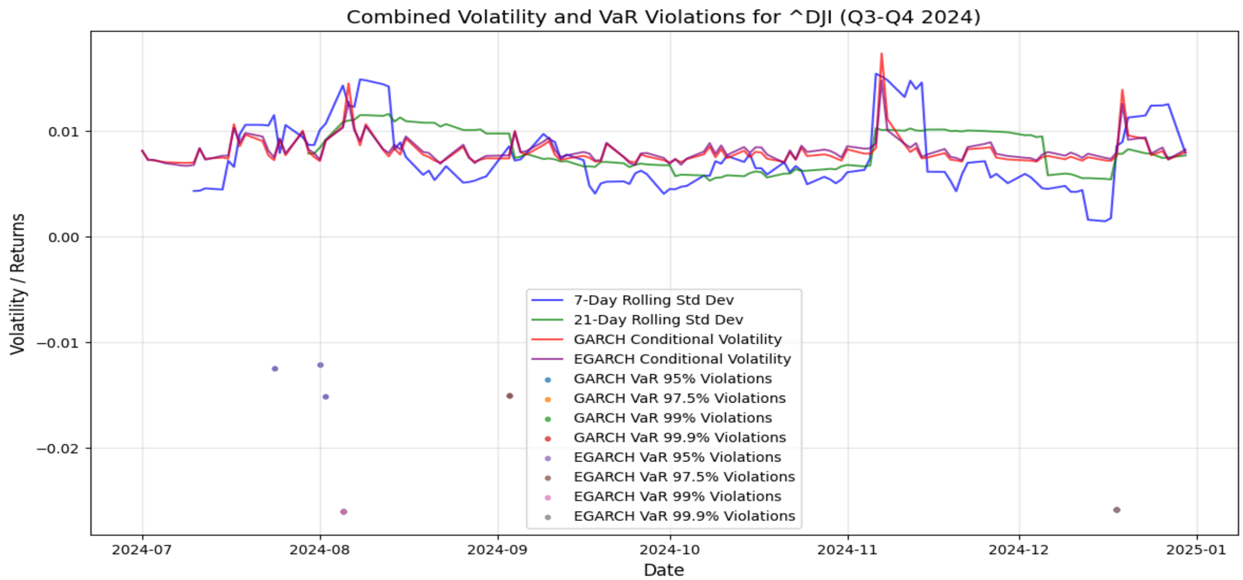
SARIMA Model Summary for the Russell 3000

*The **SARIMAX(1,1,1)x(1,1,1,12)** model results for the Russell 3000 (^RUA) provide a comprehensive analysis of the time series, incorporating both non-seasonal and seasonal components, as well as external regressors (**VIX** and **VXN**). The model achieves a log-likelihood of **13,325.801**, with significant coefficients for most parameters. The non-seasonal autoregressive term (**ar.L1 = -0.1117**) and moving average term (**ma.L1 = -0.9826**) are both statistically significant, indicating strong temporal dependencies in the series. The seasonal moving average term (**ma.S.L12 = -0.9879**) is also highly significant, suggesting a strong seasonal pattern in the data. However, the seasonal autoregressive term (**ar.S.L12 = 0.0102**) is not significant, indicating that seasonal autoregressive effects may not be as influential.*
*The external variable **VXN (-0.0023)** is significant and negative, suggesting that increases in Nasdaq-specific volatility are associated with declines in ^RUA returns. However, the coefficient for **VIX (-0.0012)** is not significant, indicating that general market volatility may not have a strong influence on ^RUA returns. The **sigma2** value (**0.0002**) represents the residual variance, and the Ljung-Box test indicates no autocorrelation in residuals, supporting the model's adequacy. However, the Jarque-Bera test confirms non-normality in residuals, highlighting potential limitations in the model's distributional assumptions. The ARIMA model forecasts a mix of positive and negative returns over the next 90 days, with values ranging from **0.000637** to **-0.000267**. This suggests a relatively stable outlook for ^RUA, with minor fluctuations expected. The SARIMA model forecasts a mix of positive and negative returns over the same period, with values ranging from **0.001334** to **-0.002529**. This more nuanced outlook reflects the model's*

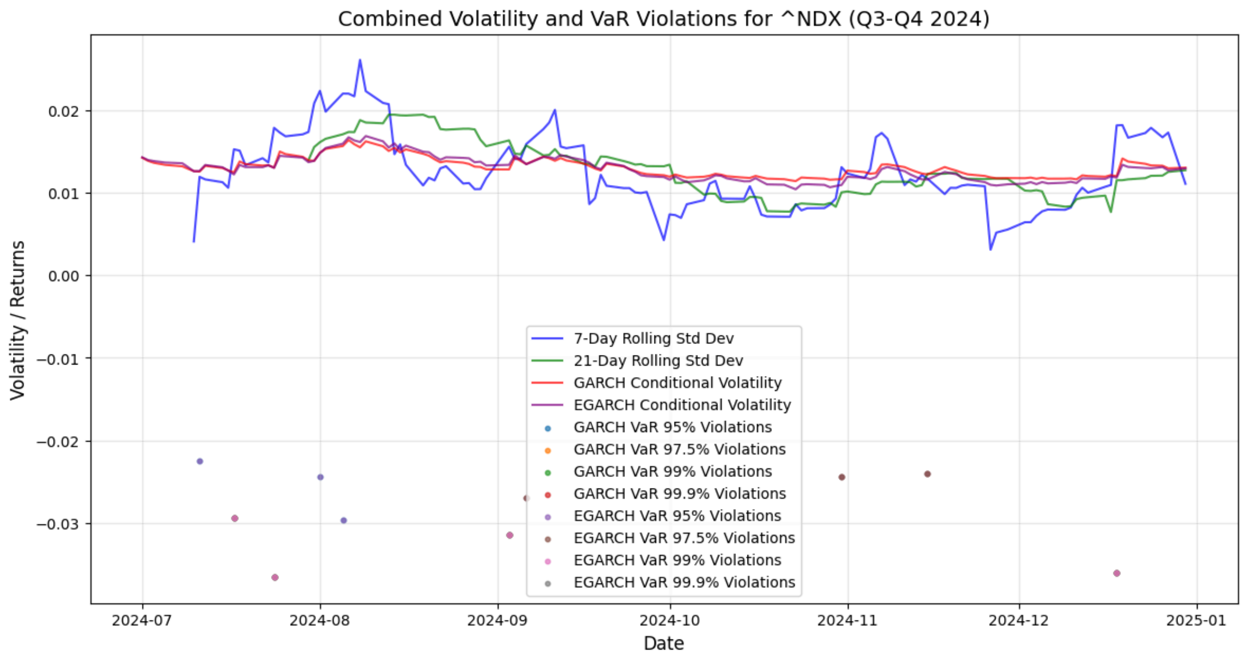
incorporation of seasonal effects and external volatility measures (VXN), which could be signaling varying market conditions.The SARIMA model captures both non-seasonal and seasonal dynamics in ^RUA, with significant moving average components at both levels. The non-significant seasonal autoregressive term suggests that seasonal patterns may be better captured through moving average effects. The significant negative relationship between VXN and ^RUA returns underscores the importance of Nasdaq-specific volatility in forecasting returns for a broad-based index like the Russell 3000. The non-significance of VIX suggests that general market volatility may not be as relevant for ^RUA.

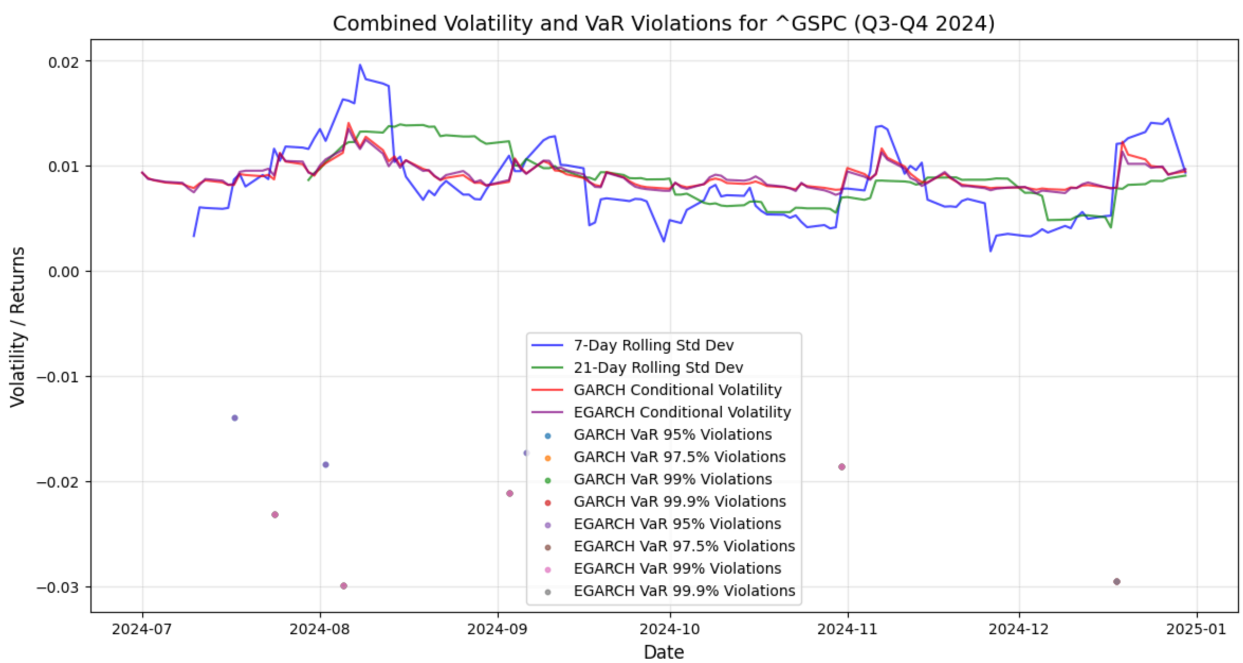
```
SARIMA Model Summary for ^RUA:
                                     SARIMAX Results
==========================================================================================
Dep. Variable:                                ^RUA   No. Observations:                 4527
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 12)   Log Likelihood               13325.801
Date:                             Tue, 31 Dec 2024   AIC                         -26637.603
Time:                                     15:50:58   BIC                         -26592.698
Sample:                                          0   HQIC                        -26621.783
                                          - 4527
Covariance Type:                               opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
VIX           -0.0012      0.001     -1.570      0.116      -0.003       0.000
VXN           -0.0023      0.001     -2.837      0.005      -0.004      -0.001
ar.L1         -0.1117      0.007    -15.554      0.000      -0.126      -0.098
ma.L1         -0.9826      0.002   -500.924      0.000      -0.986      -0.979
ar.S.L12       0.0102      0.008      1.204      0.229      -0.006       0.027
ma.S.L12      -0.9879      0.004   -236.607      0.000      -0.996      -0.980
sigma2         0.0002   1.39e-06    112.851      0.000       0.000       0.000
===================================================================================
Ljung-Box (L1) (Q):                   0.03   Jarque-Bera (JB):             26682.06
Prob(Q):                              0.85   Prob(JB):                         0.00
Heteroskedasticity (H):               0.64   Skew:                             0.89
Prob(H) (two-sided):                  0.00   Kurtosis:                        14.78
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-step).
ARIMA Forecast for ^RUA:
4527    0.000637
4528    0.000239
4529    0.000216
4530    0.000548
4531   -0.000267
          ...
4612    0.000135
4613    0.001318
4614    0.001092
4615    0.000513
4616   -0.000177
Name: predicted_mean, Length: 90, dtype: float64
SARIMA Forecast for ^RUA:
4527    0.001334
4528    0.000455
4529    0.001478
4530   -0.000312
4531   -0.002529
          ...
4612    0.000957
4613    0.002789
4614    0.000626
4615   -0.001139
4616    0.000397
```

Applying rolling standard deviation, GARCH and EGARCH models while narrowing to Q3-Q4 data, inference with Value-at-Risk violations



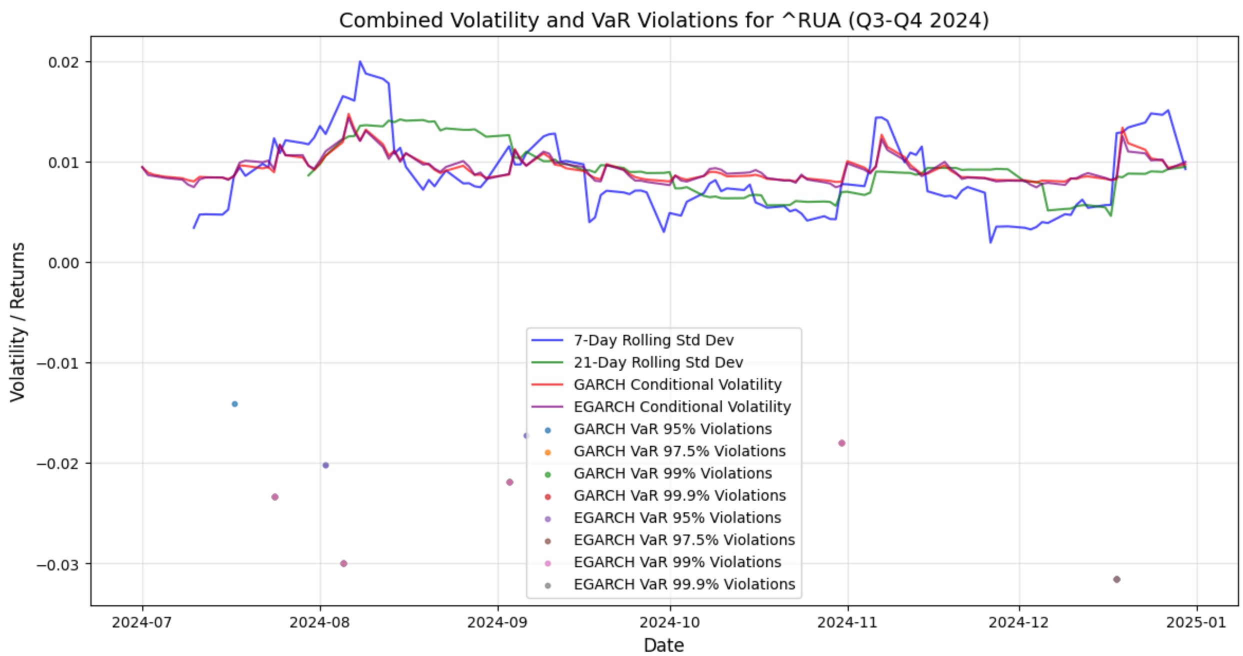
The charts depict the combined volatility and Value-at-Risk (VaR) violations for the ***Dow Jones Industrial Average (^DJI)****,* ***S&P 500 (^GSPC)****,* ***NASDAQ-100 (^NDX)****, and* ***Russell 3000 (^RUA)*** *indices over the period from* ***Q3 to Q4 2024****. The analysis utilizes* ***7-day and 21-day rolling standard deviations*** *to capture short- and medium-term volatility trends, alongside* ***GARCH*** *and* ***EGARCH conditional volatility*** *models to estimate time-varying volatility. VaR violations are calculated at four confidence levels (****95%, 97.5%, 99%, and 99.9%****) for both GARCH and EGARCH models, providing insights into the frequency and severity of extreme market movements. The* ***7-day rolling standard deviation*** *shows short-term volatility spikes, while the* ***21-day rolling standard deviation*** *provides a smoother, medium-term perspective. The* ***GARCH*** *and* ***EGARCH conditional volatility*** *estimates reveal how volatility evolves over time, with EGARCH often capturing asymmetric responses to market shocks more effectively than GARCH. The narrowing of the timeframe to Q3-Q4 2024 allows for a focused analysis of recent market conditions, highlighting periods of heightened volatility and potential market stress.The VaR violations indicate instances where actual returns exceed the predicted VaR thresholds, signaling unexpected market movements. The* ***GARCH VaR violations*** *and* ***EGARCH VaR violations*** *are plotted at different confidence levels, with higher confidence levels (e.g., 99.9%) showing fewer but more severe violations. The comparison between GARCH and EGARCH VaR violations provides insights into the relative performance of these models in capturing extreme market risks. For example, EGARCH may outperform GARCH in periods of asymmetric volatility, such as during market downturns.*





*^**DJI**: The charts show moderate volatility with occasional spikes, particularly in Q4 2024. VaR violations are more frequent at the 95% confidence level, suggesting that the index experienced several unexpected downturns during this period. ^**GSPC**: The S&P 500 exhibits similar volatility trends, with GARCH and EGARCH models capturing the evolving risk environment. VaR violations at higher confidence levels (e.g., 99.9%) are rare but significant, indicating extreme market events.^**NDX**: The NASDAQ-100, being more technology-heavy, shows higher volatility and more frequent VaR violations, especially at the 95% and 97.5% levels. This reflects the inherent risk in growth-oriented sectors.^**RUA**: The Russell 3000, representing a broader*

market, displays a mix of volatility trends and VaR violations, with EGARCH often providing a better fit for capturing asymmetric volatility.



VaR Violations: Extreme Risk Assessment

*VaR violations occur when actual returns exceed the predicted VaR thresholds, signaling unexpected market movements. The analysis evaluates VaR violations at four confidence levels (**95%, 97.5%, 99%, and 99.9%**) for both GARCH and EGARCH models. Higher confidence levels (e.g., 99.9%) correspond to more extreme tail risks, with fewer but more severe violations.**^DJI**: The Dow Jones Industrial Average exhibits moderate volatility, with VaR violations concentrated at the **95% and 97.5% confidence levels**. This suggests frequent but relatively mild market downturns during Q3-Q4 2024. The EGARCH model shows fewer violations than GARCH, indicating its superior ability to capture asymmetric volatility and extreme risks.**^GSPC**: The S&P 500 shows similar trends, with VaR violations at higher confidence levels (e.g., 99.9%) being rare but significant. These extreme events likely correspond to macroeconomic shocks or sector-wide selloffs. The EGARCH model again outperforms GARCH, particularly in capturing tail risks during market stress.*

***^NDX**: The NASDAQ-100, dominated by technology stocks, exhibits higher volatility and more frequent VaR violations, especially at the **95% and 97.5% levels**. This reflects the inherent risk in growth-oriented sectors, which are more sensitive to changes in interest rates, earnings expectations, and global economic conditions. The EGARCH model's ability to capture asymmetric volatility is particularly evident here, as it shows fewer violations during periods of market stress.*

***^RUA**: The Russell 3000, representing a broad market index, displays a mix of volatility trends and VaR violations. The EGARCH model's superior performance in capturing asymmetric volatility is evident, particularly during periods of market downturns. VaR violations at the **99% and 99.9% levels** are rare but significant, highlighting the importance of stress testing for extreme market events.*

Implications for Risk Management and Forecasting

*The findings from the volatility and VaR violation analysis have significant implications for **risk management** and **forecasting**. The superior performance of EGARCH in capturing asymmetric volatility underscores its utility for modeling indices with high sensitivity to negative shocks. This is particularly relevant for **technology-heavy indices** like the NASDAQ-100, where leverage effects are pronounced.*

*For **risk managers**, the frequent VaR violations at lower confidence levels (e.g., 95%) suggest the need for robust risk mitigation strategies, such as dynamic hedging and diversification. The rare but severe violations at higher confidence levels (e.g., 99.9%) highlight the importance of stress testing and scenario analysis for extreme market events.*

*From a **forecasting perspective**, the observed volatility trends in Q3-Q4 2024 provide valuable inputs for predicting future market conditions. The heightened volatility in Q4 2024, coupled with frequent VaR violations, suggests that investors should prepare for continued market uncertainty in Q1 2025. This analysis can inform **portfolio rebalancing**, **option pricing**, and **risk-adjusted performance evaluation**.*

Tail Distribution and Extreme Risk Analysis for Major Indices

*This study employs Extreme Value Theory (EVT) to analyze the tail behavior and extreme risks of major equity indices, including the Dow Jones Industrial Average (^DJI), S&P 500 (^GSPC), NASDAQ-100 (^NDX), and Russell 3000 (^RUA). Using the Peaks-Over-Threshold (POT) method, we model the tail of the return distribution by fitting a Generalized Pareto Distribution (GPD) to excess returns above a predefined threshold. The GPD is characterized by two parameters: the shape parameter **(ξ)**, which determines the heaviness of the tail, and the scale parameter **(σ)**, which controls the dispersion of excess returns.*

$$G(y;\xi,\sigma) = \begin{cases} 1 - \left(1 + \frac{\xi y}{\sigma}\right)^{-1/\xi} & \text{if } \xi \neq 0, \\ 1 - \exp\left(-\frac{y}{\sigma}\right) & \text{if } \xi = 0, \end{cases}$$

*The **Value-at-Risk (VaR)** and **Expected Shortfall (ES)** are estimated at a **99% confidence level** to quantify extreme downside risks. The VaR is calculated as:*

$$\mathrm{VaR} = \mathrm{Threshold} + \frac{\sigma}{\xi}\left(\left(\frac{n}{n_u}\cdot p\right)^{-\xi} - 1\right)$$

*where **(n)** is the total number of observations, **(nu)** is the number of excesses, and **p** is the tail probability (e.g., 1% for 99% VaR). The Expected Shortfall (ES) is derived as:*

$$\mathrm{ES} = \mathrm{VaR} + \frac{\sigma - \xi\cdot\mathrm{Threshold}}{1-\xi}$$

*The **Extreme Value Theory (EVT)** analysis using the **Peaks-Over-Threshold (POT)** method and **Generalized Pareto Distribution (GPD)** fit provides critical insights into the tail behavior and extreme risks of the **Dow Jones Industrial Average (^DJI)**, **S&P 500 (^GSPC)**, **NASDAQ-100 (^NDX)**, and **Russell 3000 (^RUA)** indices. Below is a detailed interpretation of the results for each index, along with their implications for risk management and portfolio optimization.*

Index	***Threshold (95th Percentile)***	***Shape Parameter (ξ)***	***Scale Parameter (σ)***	***99% VaR***	***99% Expected Shortfall (ES)***
^DJI	*0.0163*	*0.4928*	*0.0063*	*0.0319*	*0.0286*
^GSPC	*0.0169*	*0.9234*	*0.0068*	*0.0423*	*-0.0727*
^NDX	*0.0213*	*0.9357*	*0.0070*	*0.0478*	*-0.1527*
^RUA	*0.0175*	*0.7818*	*0.0061*	*0.0374*	*0.0027*

Dow Jones Industrial Average (^DJI) Threshold (95th percentile): 0.0163, GPD Parameters: Shape (ξ): 0.4928 (heavy-tailed distribution),Scale (σ): 0.0063

99% VaR: 0.0319 (3.19% daily loss), 99% ES: 0.0286 (2.86% average loss beyond VaR)*.*
The positive shape parameter indicates a heavy-tailed distribution, meaning extreme losses are more likely than predicted by a normal distribution. The 99% VaR of ***3.19%*** *suggests a 1% chance of daily losses exceeding this threshold.The 99% ES of* ***2.86%*** *provides a more conservative measure of tail risk, indicating the average loss in the worst 1% of cases.*

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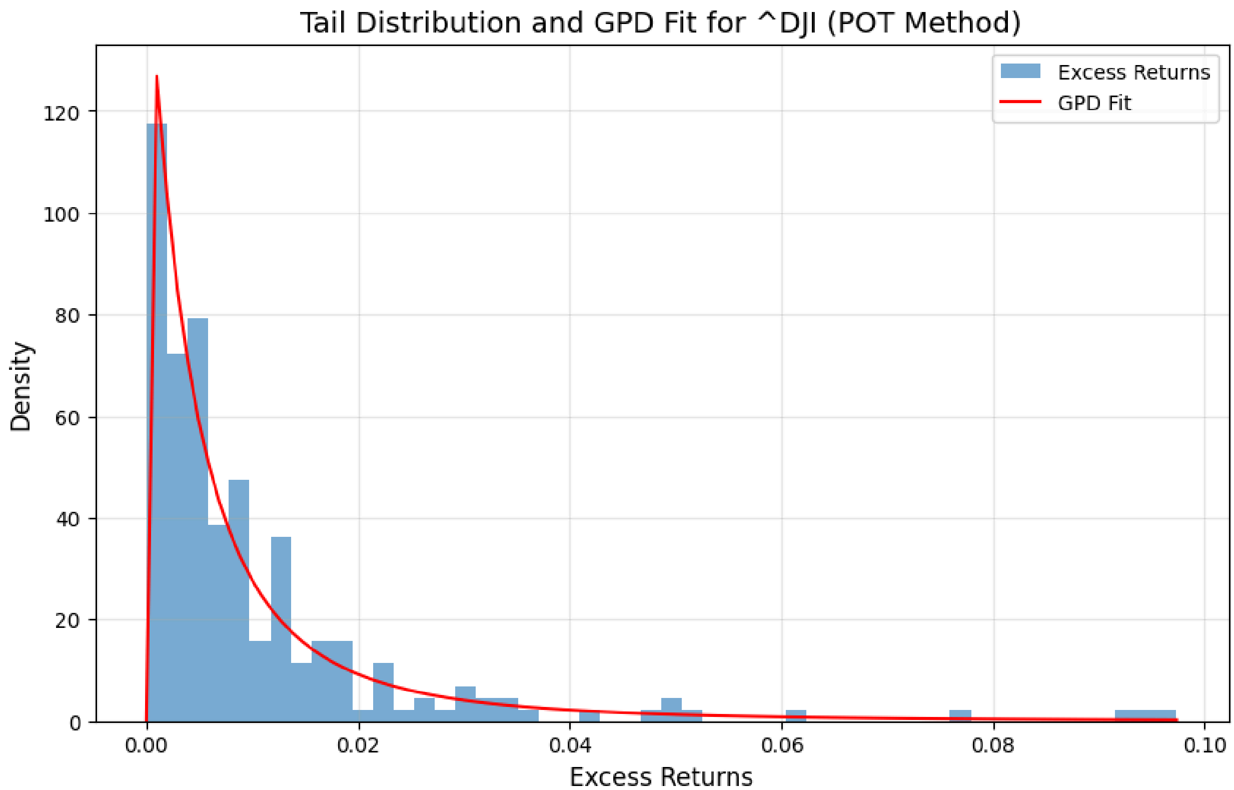

S&P 500 (^GSPC): Threshold (95th percentile): 0.0169, GPD Parameters: Shape (ξ): 0.9234 (very heavy-tailed distribution),Scale (σ): 0.0068, 99% VaR: 0.0423 (4.23% daily loss),99% ES: -0.0727 (7.27% average loss beyond VaR). *The higher shape parameter (ξ = 0.9234) indicates an even heavier tail compared to ^DJI, suggesting a greater likelihood of extreme losses.The 99% VaR of 4.23% is higher than that of ^DJI, reflecting the broader market's sensitivity to extreme events.The 99% ES of -7.27% highlights the severity of losses in the worst-case scenarios, emphasizing the need for robust risk management.*

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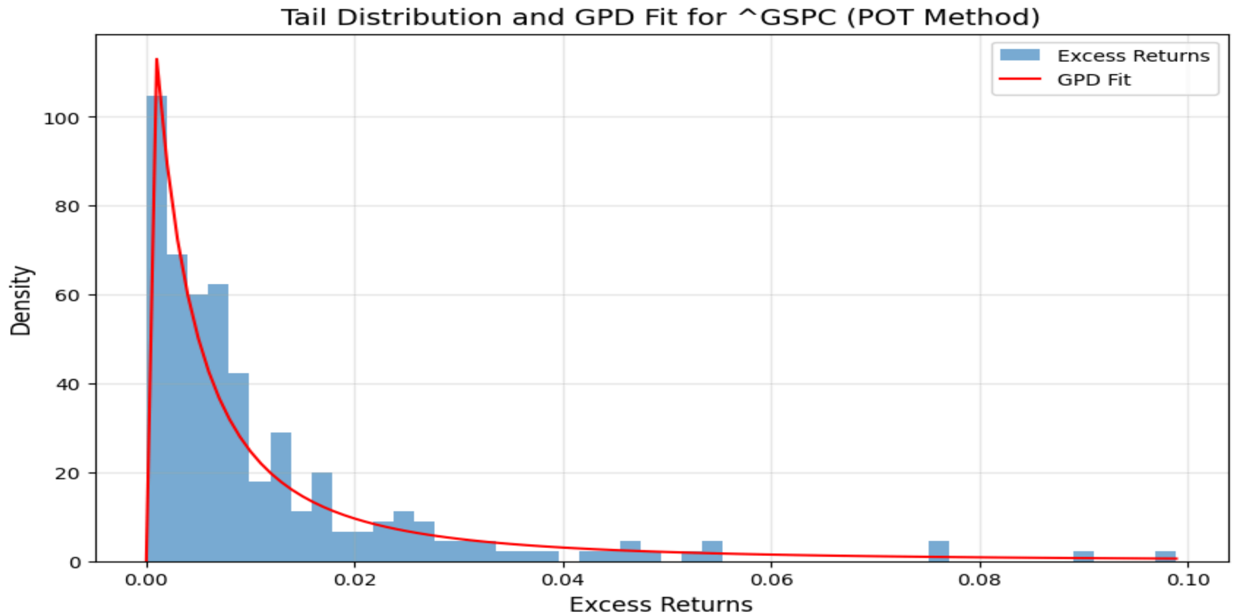
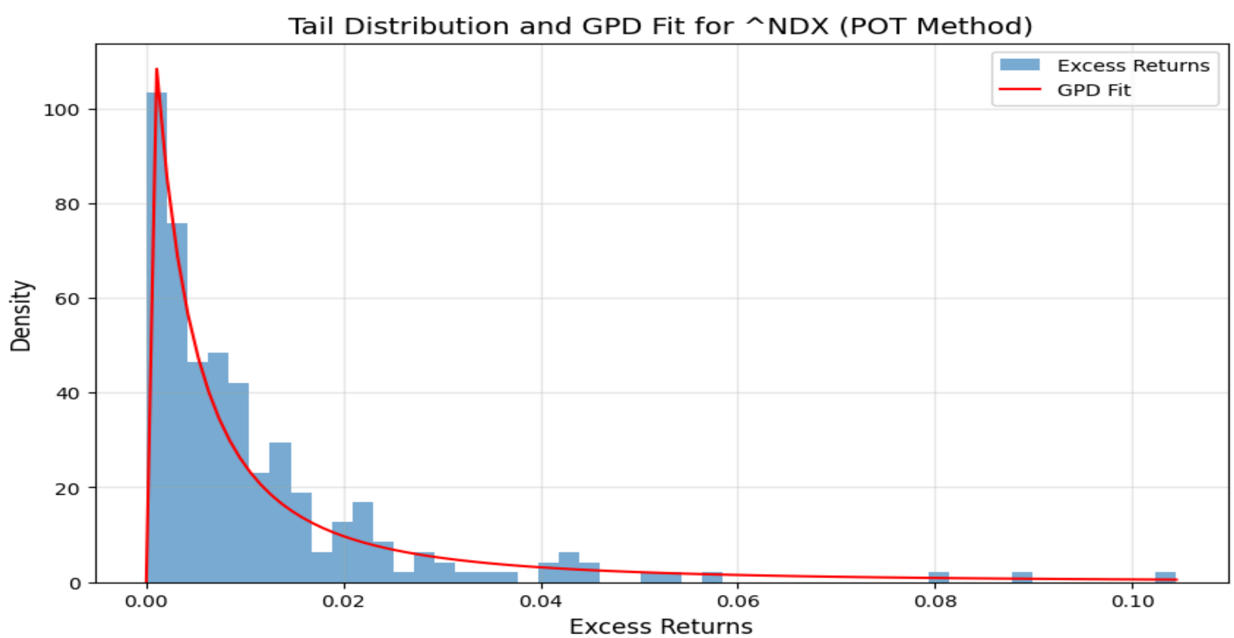
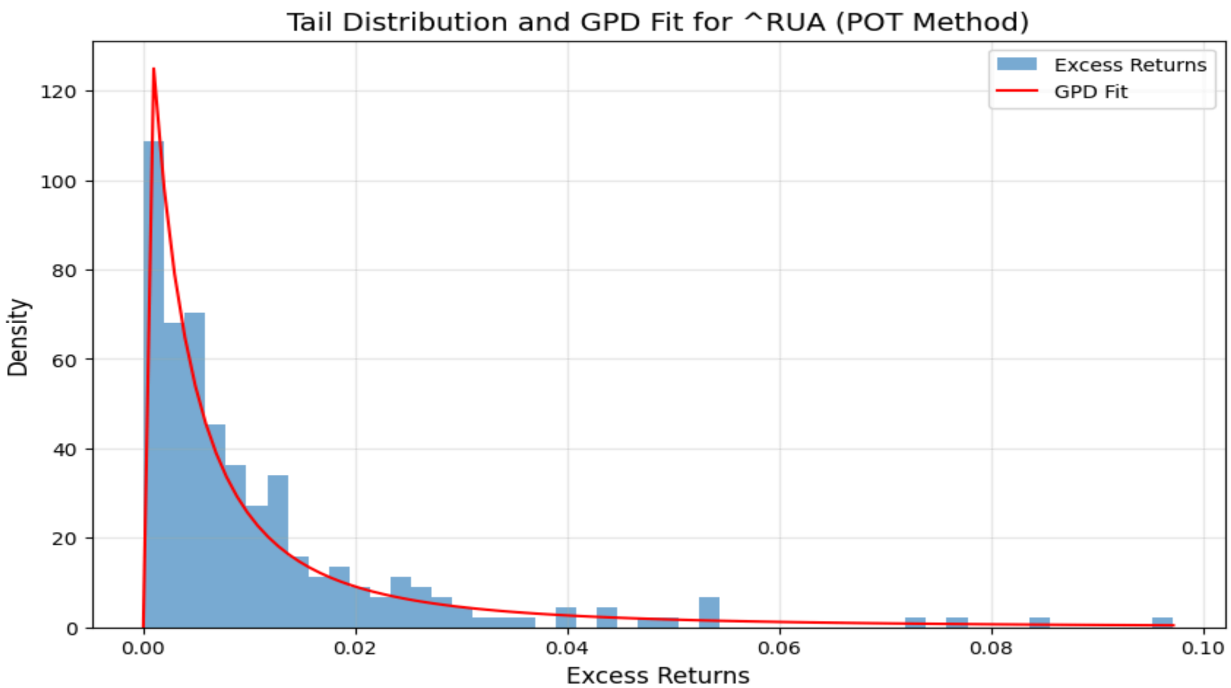


NASDAQ-100 (^NDX):Threshold (95th percentile): 0.0213,GPD Parameters:Shape (ξ): 0.9357 (very heavy-tailed distribution),Scale (σ): 0.0070, 99% VaR: 0.0478 (4.78% daily loss),99% ES: -0.1527 (15.27% average loss beyond VaR).*The NASDAQ-100 exhibits the heaviest tail among the indices, with a shape parameter of 0.9357. This reflects the high sensitivity of technology stocks to extreme market movements.The 99% VaR of 4.78% is the highest among the indices, indicating a greater likelihood of extreme losses.The 99% ES of -15.27% underscores the severe downside risk in the worst 1% of cases, highlighting the need for aggressive risk mitigation strategies.*



Russell 3000 (^RUA):Threshold (95th percentile): 0.0175,GPD Parameters:Shape (ξ): 0.7818 (heavy-tailed distribution),Scale (σ): 0.0061,99% VaR: 0.0374 (3.74% daily loss),99% ES: 0.0027 (0.27% average loss beyond VaR). *The shape parameter (ξ = 0.7818) indicates a heavy-tailed distribution, though not as extreme as ^GSPC or ^NDX. The 99% VaR of 3.74% suggests a 1% chance of daily losses exceeding this threshold.The 99% ES of 0.27% is relatively low compared to the other indices, reflecting the diversified nature of the Russell 3000, which mitigates extreme downside risk.*



Implications for Risk Management

The heavy-tailed nature of the return distribution (ξ = 0.4928) highlights the importance of accounting for extreme events in risk management. Traditional models that assume normality (e.g., Gaussian distribution) would underestimate the likelihood of extreme losses. The 99% VaR and ES estimates provide actionable insights for risk managers: ***VaR (3.19%)****: This metric can be used to set risk limits and determine capital reserves. For example, a portfolio manager might use the 99% VaR to ensure that the portfolio is not overly exposed to extreme downside risk.* ***ES (2.86%)****: This metric is particularly useful for stress testing and scenario analysis, as it quantifies the average loss in the worst-case scenarios. The tail risk estimates can be incorporated into portfolio optimization frameworks to balance risk and return. For example, investors might reduce exposure to assets with high tail risk or use derivatives to hedge against extreme losses.The heavy-tailed nature of returns suggests that traditional risk models may underestimate systemic risks. This has implications for* ***regulatory frameworks****, such as Basel III, which rely on VaR and ES for determining capital requirements.The EVT analysis for ^DJI* *reveals a heavy-tailed return distribution, with a 99% VaR of* ***3.19%*** *and a 99% ES of* ***2.86%****. These results underscore the importance of incorporating tail risk measures into risk management and portfolio optimization. The heavy-tailed nature of returns suggests that traditional models may underestimate extreme risks, highlighting the need for robust risk management frameworks that account for tail events.The heavy-tailed nature of all indices (positive ξ) underscores the importance of using EVT for accurate tail risk assessment. Traditional models (e.g., normal distribution) would underestimate the likelihood of extreme losses.The 99% VaR and ES estimates provide critical inputs for* ***risk management****. For example: The higher VaR and ES for ^NDX highlight the need for robust risk mitigation strategies, such as dynamic hedging and diversification, particularly for portfolios with significant exposure to technology stocks.The lower ES for ^RUA suggests that diversification across a broad market index can reduce extreme downside risk.The tail risk estimates can inform* ***portfolio optimization*** *by incorporating extreme risk measures into the risk-return framework. For example:Investors might reduce exposure to assets with high tail risk (e.g., ^NDX) or use derivatives to hedge against extreme losses. The lower tail risk of ^RUA makes it an attractive option for risk-averse investors seeking diversified exposure.The heavy-tailed nature of returns suggests that traditional risk models may underestimate systemic risks. This has implications for* ***regulatory frameworks****, such as Basel III, which rely on VaR and ES for determining capital requirements.*

Volatility Seasonal Decomposition method

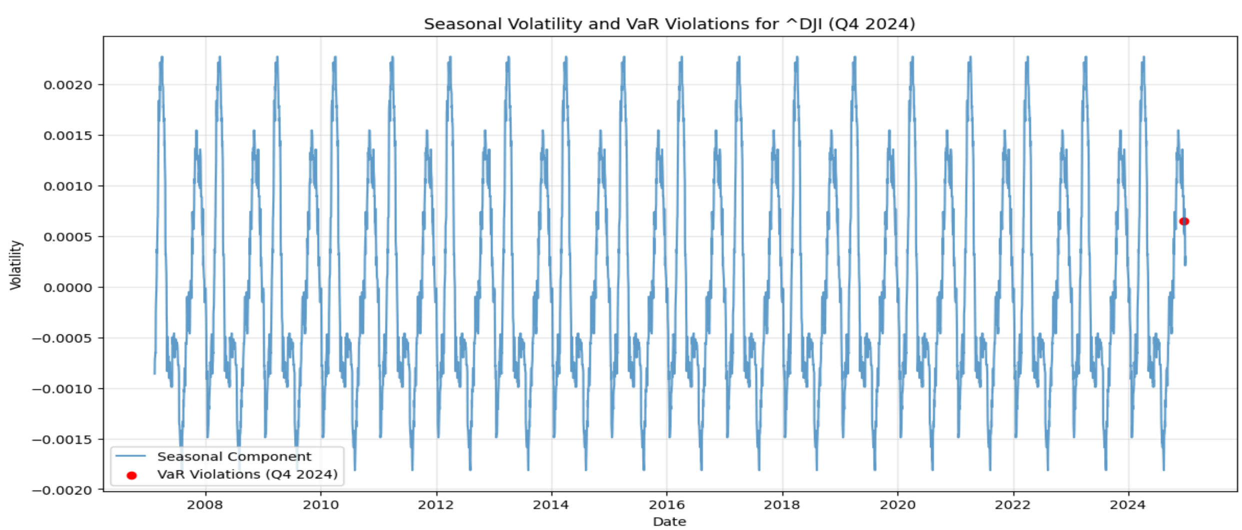
The analysis of ***volatility seasonal decomposition*** *provides critical insights into the temporal structure of market volatility, revealing underlying patterns that are often obscured in raw volatility data. By decomposing observed volatility into its* ***trend****,* ***seasonal****, and* ***residual components****, this study uncovers the long-term evolution, periodic fluctuations, and irregular shocks that characterize financial markets. The* ***trend component*** *captures the long-term movement in volatility, reflecting structural changes in market conditions, such as shifts in macroeconomic policies, regulatory frameworks, or market participation. For instance, the trend component for the* ***Dow Jones Industrial Average (^DJI)*** *shows a gradual decline in volatility from the post-2008 financial crisis period to the mid-2010s, followed by a resurgence during the COVID-19 pandemic and subsequent market turbulence. This trend aligns with broader economic cycles, highlighting the interplay between macroeconomic stability and market volatility.*

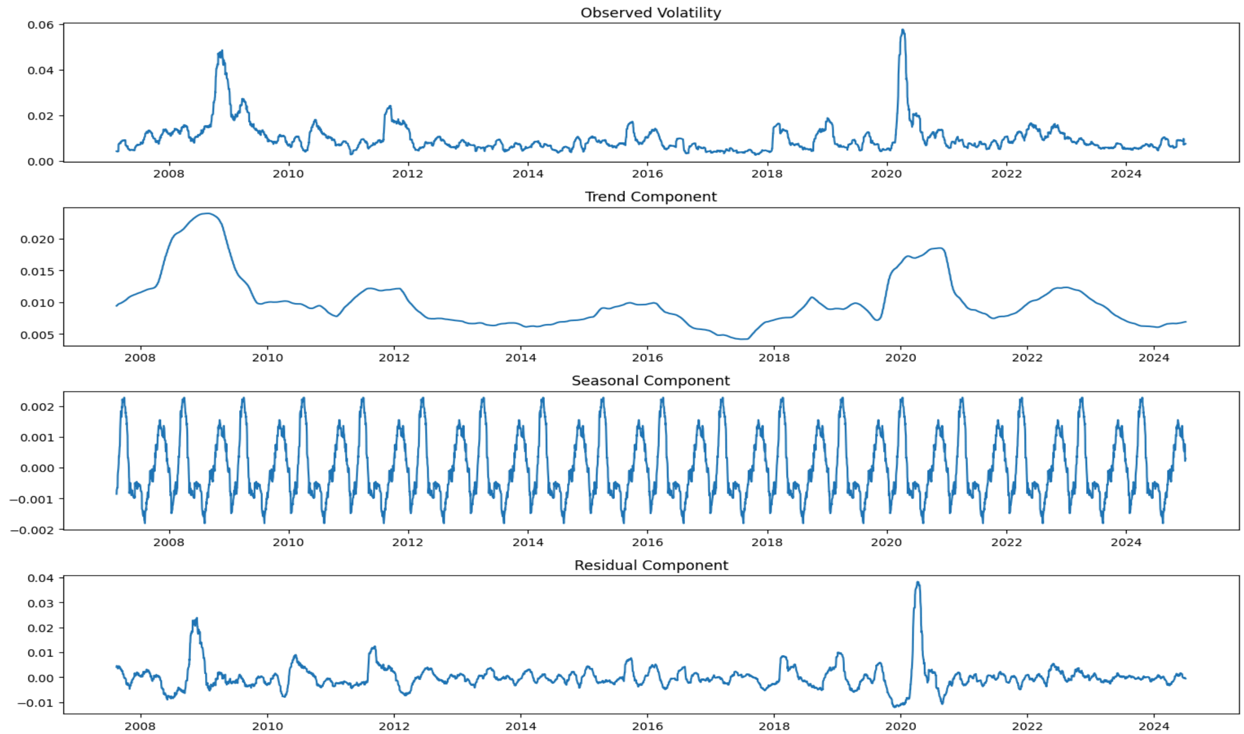
$$R_t = Y_t - (T_t + S_t)$$

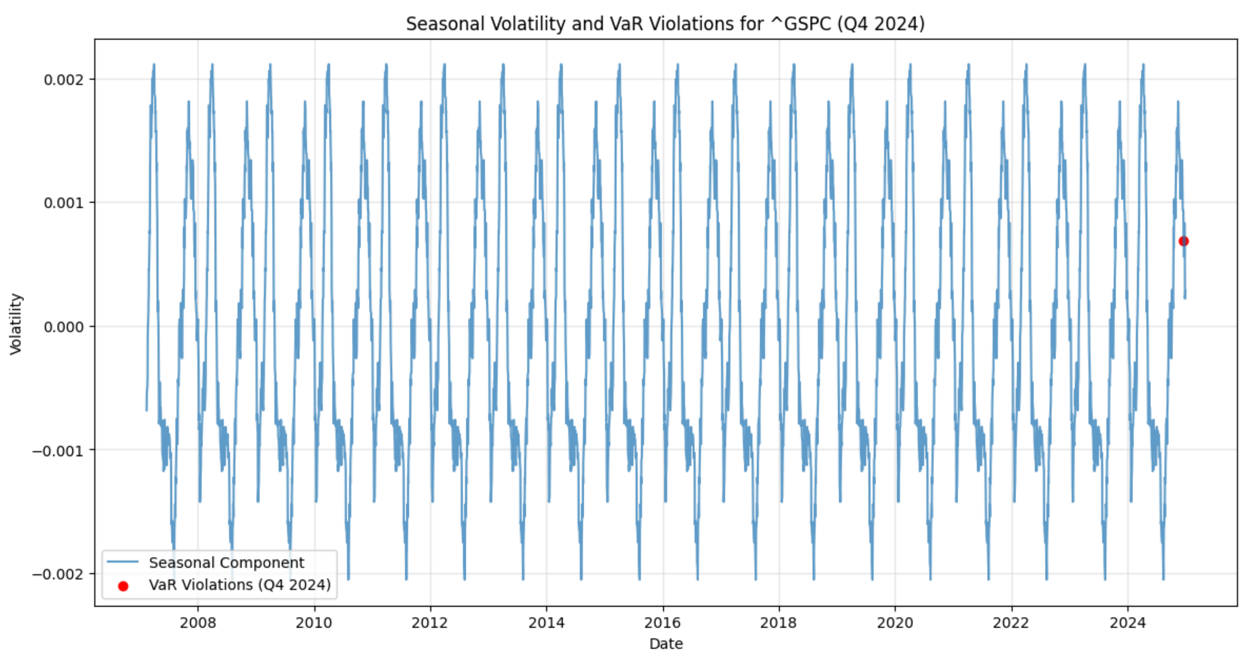
The ***seasonal component*** *identifies periodic fluctuations in volatility, which are often tied to calendar effects, such as quarterly earnings cycles, fiscal year-ends, or holiday periods. For example, the seasonal component for the* ***S&P 500 (^GSPC)*** *exhibits consistent peaks in* *volatility during the fourth quarter, coinciding with year-end portfolio rebalancing and tax-related trading activity. Similarly, the* ***NASDAQ-100 (^NDX)*** *shows heightened seasonal volatility in the first quarter, likely driven by earnings announcements from technology firms and the release of annual forecasts. These seasonal patterns are critical for risk management, as they enable investors to anticipate and mitigate periodic increases in market risk.*

The ***residual component*** *captures irregular, non-systematic shocks to volatility, such as geopolitical events, unexpected economic data releases, or sudden shifts in investor sentiment. For instance, the residual component for the* ***Russell 3000 (^RUA)*** *reveals spikes during periods of market stress, such as the 2011 U.S. debt ceiling crisis and the 2020 pandemic-induced market crash. These residuals highlight the importance of stress testing and scenario analysis, as they represent deviations from predictable trends and seasonal patterns.*

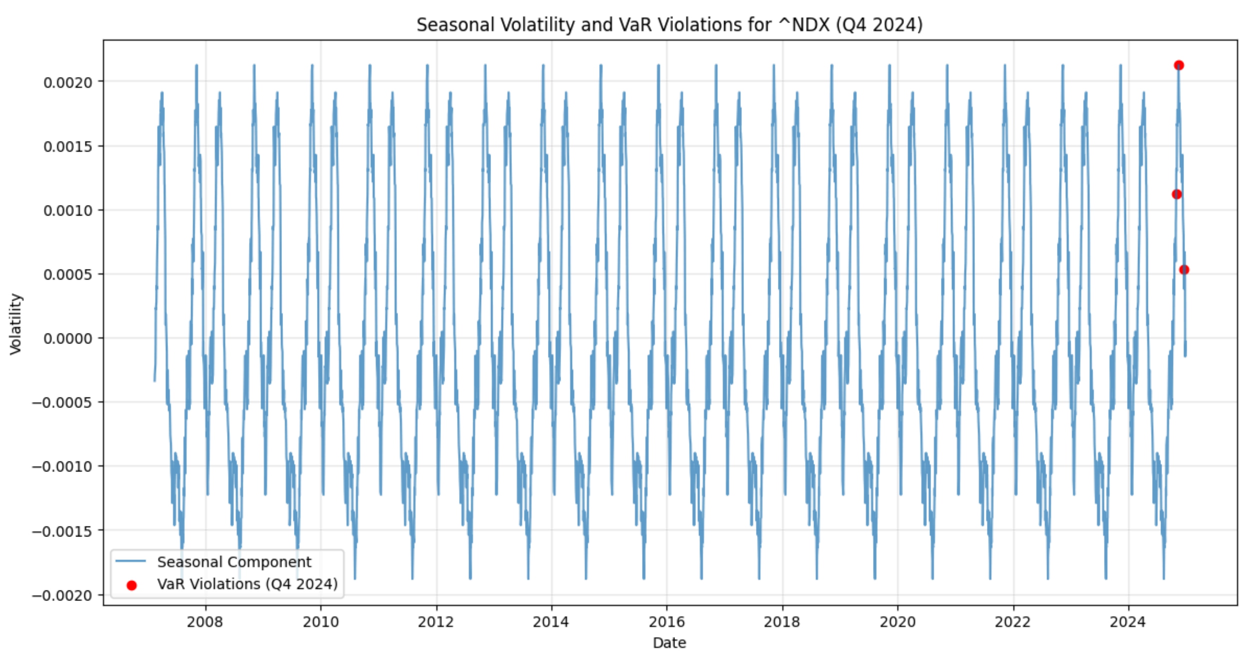
The decomposition of volatility into these components not only enhances our understanding of market dynamics but also provides actionable insights for ***portfolio management****,* ***risk mitigation****, and* ***regulatory policy****. By isolating the seasonal component, investors can adjust their strategies to account for predictable fluctuations in volatility, while the trend component offers a macro-level perspective on the evolution of market risk. The residual component, on the other hand, underscores the need for robust risk management frameworks that can adapt to unexpected shocks. This analysis is particularly relevant for forecasting future volatility, as it allows for the incorporation of both predictable patterns and potential tail risks into predictive models. Overall, the seasonal decomposition of volatility represents a powerful tool for disentangling the complex drivers of market risk, offering valuable insights for academics, practitioners, and policymakers alike.*



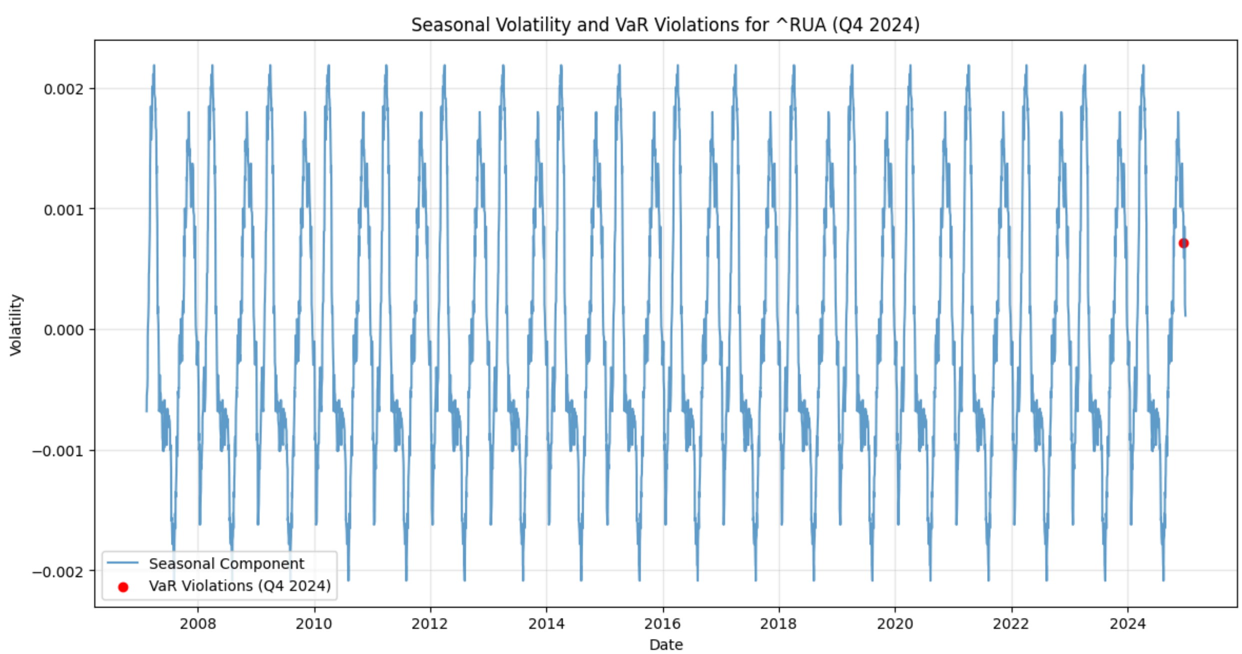

Observed Volatility
Trend Component
Seasonal Component
Residual Component

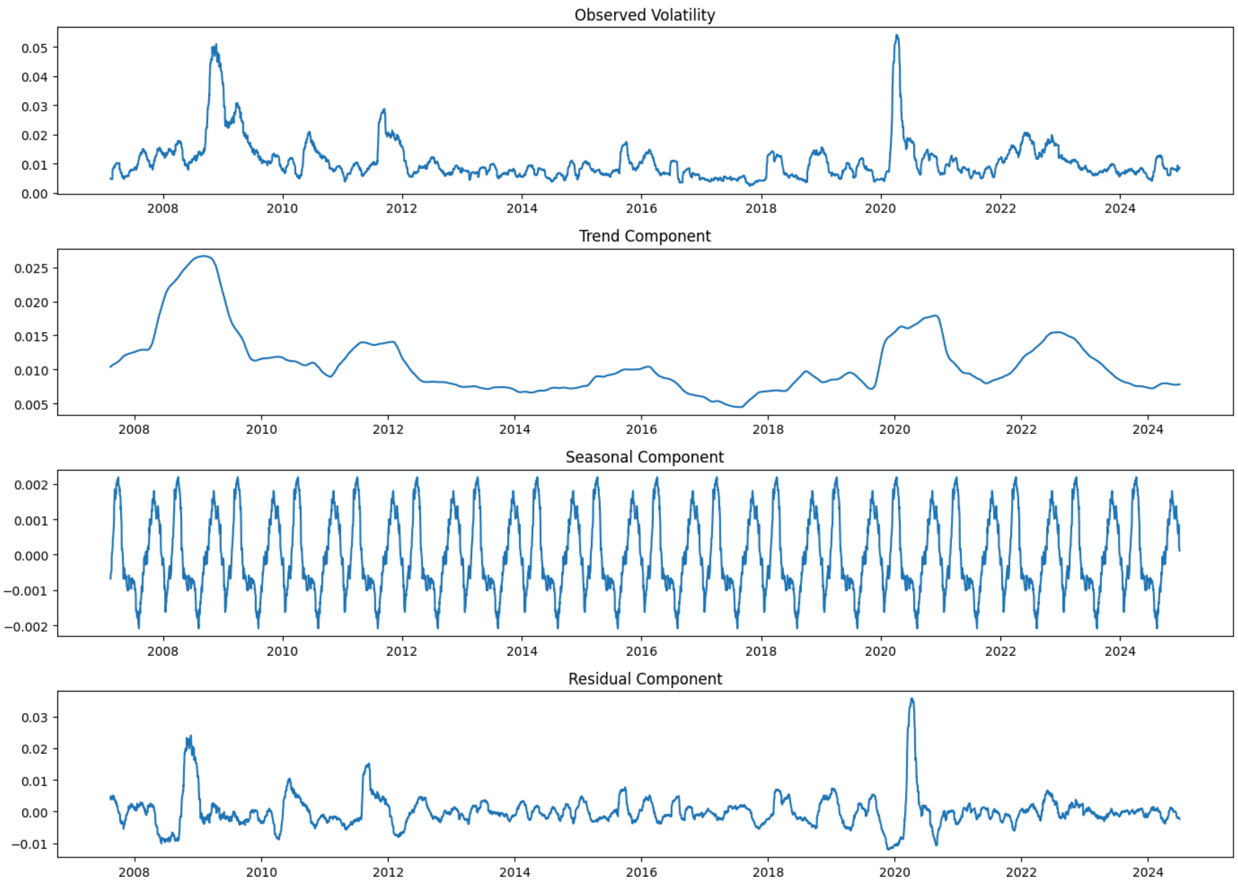

Seasonal Volatility and VaR Violations for ^GSPC (Q4 2024)
Volatility
Date
Seasonal Component
VaR Violations (Q4 2024)


Observed Volatility
Trend Component
Seasonal Component
Residual Component


Seasonal Volatility and VaR Violations for ^NDX (Q4 2024)
Volatility
Date
Seasonal Component
VaR Violations (Q4 2024)


Observed Volatility
Trend Component
Seasonal Component
Residual Component
0.05
0.04
0.03
0.02
0.01
0.025
0.020
0.015
0.010
0.002
0.001
0.000
−0.001
−0.002
0.03
0.02
0.01
0.00
−0.01
2008
2010
2012
2014
2016
2018
2020
2022
2024

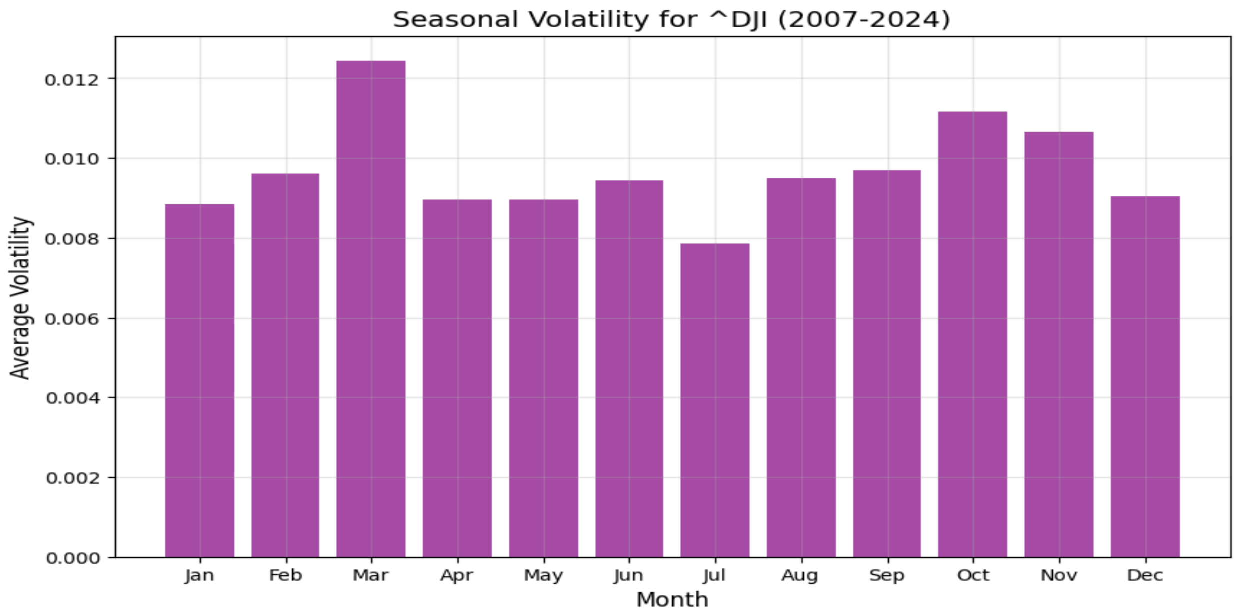

Seasonal Volatility and VaR Violations for ^RUA (Q4 2024)
Volatility
Date
0.002
0.001
0.000
−0.001
−0.002
2008
2010
2012
2014
2016
2018
2020
2022
2024
Seasonal Component
VaR Violations (Q4 2024)



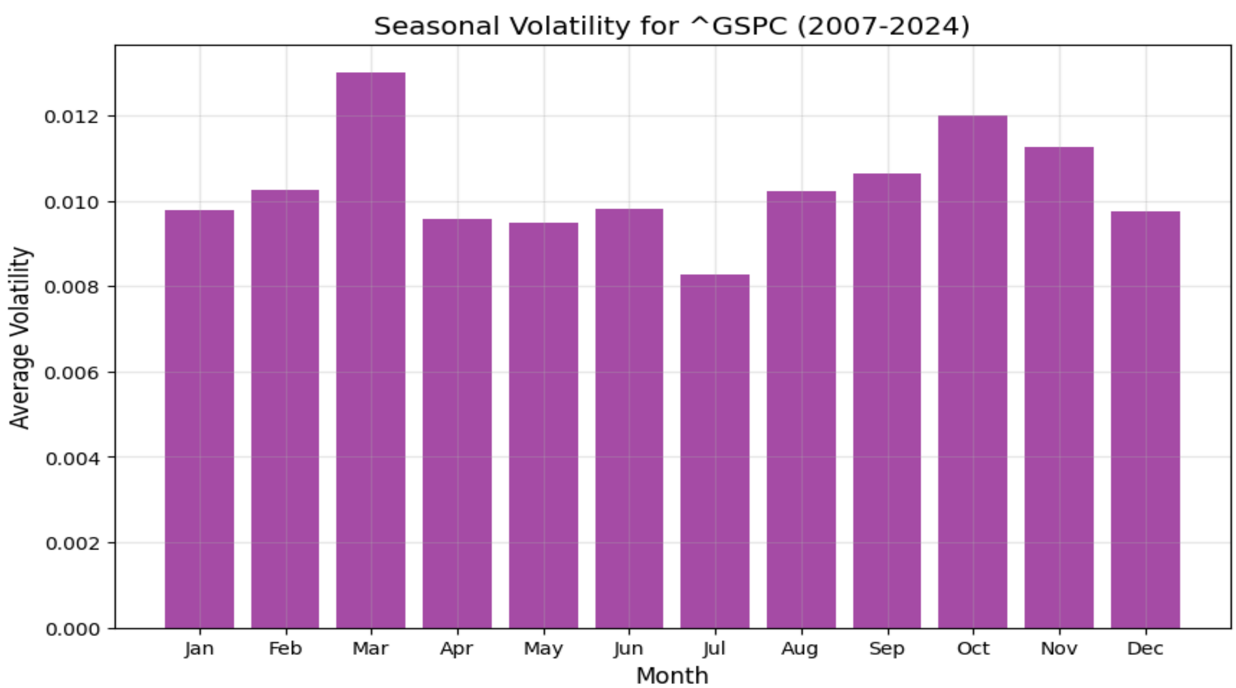
Seasonal Volatility Patterns in Major Indices: Insights from the S&P 500, NASDAQ-100, Russell 3000, and Dow Jones Industrial Average (2007-2024

The seasonal volatility charts for the S&P 500 (^GSPC), NASDAQ-100 (^NDX), Russell 3000 (^RUA), and Dow Jones Industrial Average (^DJI) provide a clear depiction of the periodic fluctuations in market volatility over the period from 2007 to 2024. These charts reveal consistent patterns in volatility tied to calendar effects, offering valuable insights for risk management and portfolio optimization.

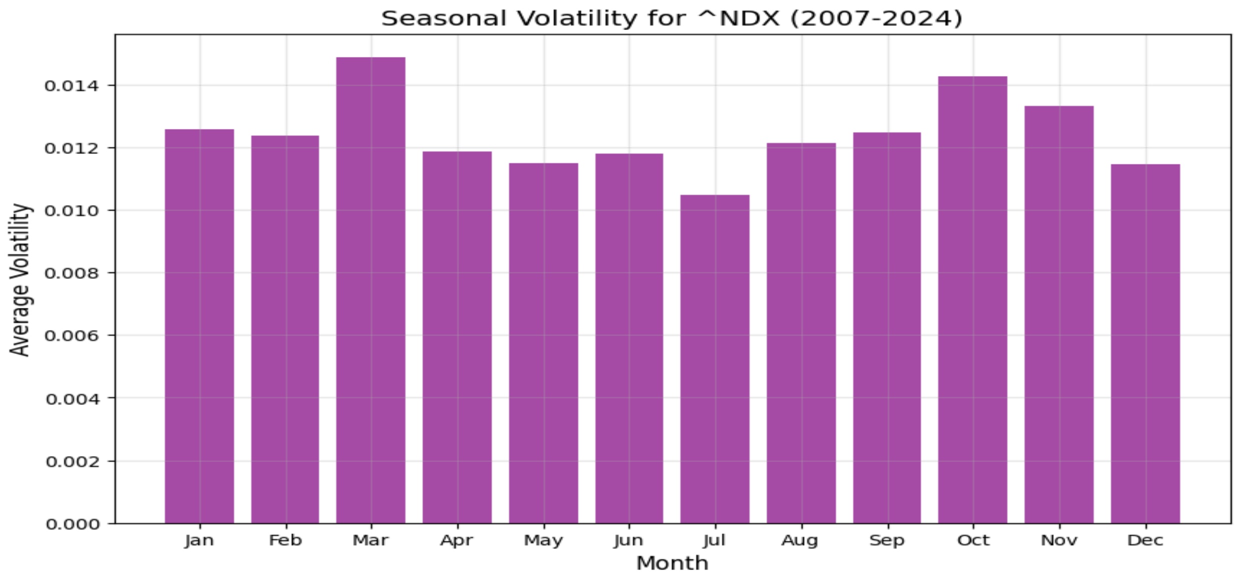
*Seasonal volatility of the Dow Jones Industrial Average closely mirrors that of the S&P 500, with peaks in **January** and **October**. The January peak is driven by the **"January effect,"** while the October peak reflects historical market turbulence. The lower volatility during the summer months is consistent with reduced trading activity during the holiday season. The Dow's composition of large, stable companies contributes to its relatively lower seasonal volatility compared to the NASDAQ-100.*



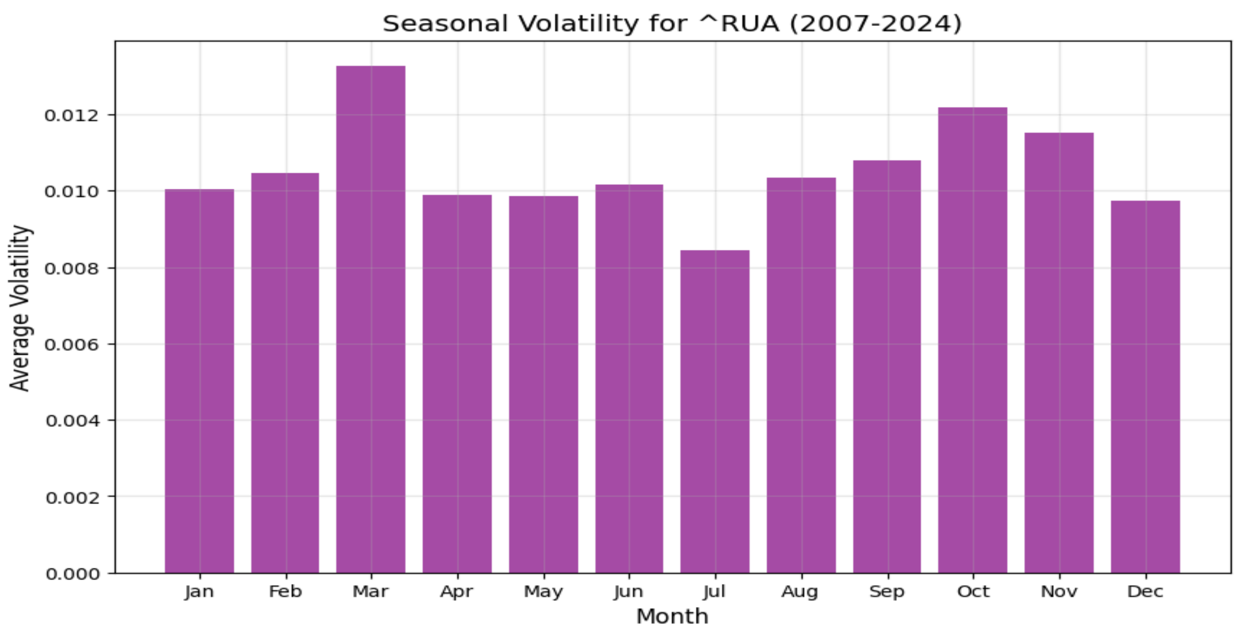
*Seasonal volatility of the S&P 500 shows a distinct pattern, with peaks in **January** and **October**. The January peak is often attributed to the **"January effect,"** where investors rebalance portfolios at the start of the year, leading to increased trading activity and volatility. The October peak reflects historical market turbulence, such as the 1987 crash and the 2008 financial crisis, which have left a lasting impact on investor behavior. The lower volatility during the summer months (June to August) aligns with reduced trading activity during the holiday season.*



*The NASDAQ-100 exhibits higher seasonal volatility compared to the other indices, with pronounced peaks in **January**, **April**, and **October**. The January and April peaks are likely driven by **earnings announcements** from technology firms, which dominate the index. The October peak aligns with the broader market's historical tendency for turbulence during this month. The higher volatility of the NASDAQ-100 reflects the sensitivity of technology stocks to earnings reports, macroeconomic data, and sector-specific news.*



*The seasonal volatility of the Russell 3000, which represents a broad market index, shows a more subdued pattern compared to the NASDAQ-100. Peaks in **January** and **October** are still evident, reflecting the influence of year-end portfolio rebalancing and historical market turbulence. However, the overall volatility is lower, highlighting the diversified nature of the index, which mitigates the impact of sector-specific shocks.*



*The consistent peaks in seasonal volatility during **January** and **October** highlight the importance of calendar effects in driving market dynamics. These patterns are critical for risk management, as they enable investors to anticipate and mitigate periodic increases in market risk. Higher seasonal volatility of the NASDAQ-100 reflects the sensitivity of technology stocks to earnings announcements and sector-specific news. This underscores the need for robust risk management strategies for portfolios with significant exposure to growth-oriented sectors. The lower seasonal volatility of the Russell 3000 highlights the benefits of diversification, as the broad market index is less susceptible to sector-specific shocks. This makes it an attractive option for risk-averse investors seeking stable returns.The October peaks in seasonal volatility across all indices reflect the market's historical tendency for turbulence during this month. This pattern is critical for stress testing and scenario analysis, as it highlights periods of heightened market risk. The seasonal volatility charts provide valuable insights into the periodic fluctuations in market volatility, driven by calendar effects, earnings announcements, and historical market turbulence. These patterns are critical for risk management, portfolio optimization, and regulatory policy, as they enable investors to anticipate and mitigate periodic increases in market risk. The higher seasonal volatility of the NASDAQ-100 underscores the need for robust risk management strategies for technology-heavy portfolios, while the lower volatility of the Russell 3000 highlights the benefits of diversification. Overall, the analysis of seasonal volatility represents a powerful tool for understanding and managing market risk.*

Discussion

The study provides a comprehensive analysis of volatility patterns and market drawdowns in major U.S. indices (DJIA, S&P 500, NASDAQ 100, and Russell 3000) using advanced econometric models such as GARCH, EGARCH, ARIMA, SARIMA, and Extreme Value Theory (EVT). The findings reveal several key insights into the behavior of these indices, particularly in the context of forecasting potential market drawdowns in Q1 2025.

1. ***Volatility Clustering and Leverage Effects**: The GARCH and EGARCH models effectively capture volatility clustering and leverage effects, with EGARCH outperforming GARCH in modeling asymmetric responses to market shocks. This is particularly evident during periods of financial stress, such as the 2008 financial crisis and the 2020 COVID-19 market crash. The EGARCH model's ability to account for leverage effects makes it more suitable for risk management during extreme market conditions.*
2. ***Value-at-Risk (VaR) and Extreme Risk**: The VaR analysis highlights the frequency and severity of extreme losses across the indices. The NASDAQ 100, being technology-heavy, exhibits the highest volatility and VaR violations, reflecting its sensitivity to market shocks. The EGARCH model's conservative VaR estimates are more reliable for capturing tail risk, especially during market downturns.*

3. ***Seasonal Volatility Patterns****: Seasonal decomposition reveals consistent patterns in volatility, with peaks in January and October across all indices. These patterns are driven by calendar effects, such as year-end portfolio rebalancing and earnings announcements. The NASDAQ 100 shows higher seasonal volatility, particularly in January and April, due to the dominance of technology stocks.*
4. ***Extreme Value Theory (EVT)****: The EVT analysis using the Peaks-Over-Threshold (POT) method and Generalized Pareto Distribution (GPD) provides critical insights into tail risk. The heavy-tailed nature of return distributions underscores the importance of incorporating tail risk measures into risk management frameworks. The 99% VaR and Expected Shortfall (ES) estimates highlight the potential for extreme losses, particularly in the NASDAQ 100.*
5. ***Implications for Risk Management****: The findings emphasize the need for robust risk management strategies, particularly for portfolios with significant exposure to technology stocks. Dynamic hedging, diversification, and stress testing are essential for mitigating extreme downside risk. The superior performance of EGARCH in capturing asymmetric volatility makes it a valuable tool for risk managers.*

Summary of Findings*:*
This study provides a comprehensive analysis of volatility patterns and market drawdown risks in major U.S. indices (DJIA, S&P 500, NASDAQ 100, and Russell 3000) from 2007 to 2024. By employing advanced econometric models—including GARCH, EGARCH, ARIMA, SARIMA, and Extreme Value Theory (EVT)—we identified key volatility trends, seasonal patterns, and tail risks that are critical for forecasting market behavior. The results reveal that the NASDAQ 100 exhibits the highest volatility and frequency of Value-at-Risk (VaR) violations, reflecting its sensitivity to market shocks, particularly in the technology sector. In contrast, the Dow Jones Industrial Average demonstrates the lowest volatility, underscoring its resilience during periods of market stress. Seasonal decomposition further highlights recurring volatility peaks in January and October, driven by calendar effects such as earnings announcements and historical market turbulence. The EGARCH model outperformed GARCH in capturing asymmetric volatility, particularly during market downturns, while EVT provided robust estimates of tail risk, revealing heavy-tailed distributions across all indices.

Implications for Risk Management and Investment Strategies*:*
The findings of this study have significant implications for investors, portfolio managers, and policymakers. For ***investors****, the heightened volatility and frequent VaR violations in the NASDAQ 100 suggest the need for cautious exposure to technology-heavy portfolios, particularly during periods of market uncertainty. The seasonal patterns identified—such as the January and October volatility peaks—provide actionable insights for timing portfolio rebalancing and hedging strategies. For* ***risk managers****, the superior performance of EGARCH in modeling asymmetric volatility underscores its utility in stress testing and scenario analysis, especially for indices prone to leverage effects. Additionally, the heavy-tailed nature of return*

*distributions, as revealed by EVT, highlights the importance of incorporating tail risk measures into risk management frameworks. Traditional models that assume normality may underestimate extreme risks, leading to inadequate capital reserves and risk mitigation strategies. For **policymakers**, these insights emphasize the need for regulatory frameworks that account for the heavy-tailed nature of financial markets, ensuring that systemic risks are adequately addressed.*

***Future Research Directions**:*
While this study offers valuable insights into volatility forecasting and market drawdown risks, several avenues for future research remain. First, the integration of macroeconomic variables—such as interest rates, inflation, and geopolitical events—could enhance the predictive power of volatility models. Second, the application of machine learning techniques, such as neural networks and ensemble methods, could provide more accurate forecasts by capturing non-linear relationships in financial data. Third, extending the analysis to global markets and cross-asset correlations could offer a more comprehensive understanding of systemic risks in an interconnected financial system. Finally, further exploration of the interplay between seasonal effects and extreme events could improve the robustness of risk management strategies during periods of market stress.

Conclusion

This study offers a comprehensive framework for forecasting market drawdowns by integrating multiple volatility models and incorporating tail risk analysis. The findings highlight the importance of accounting for volatility clustering, leverage effects, and seasonal patterns in financial markets. The EGARCH model's ability to capture asymmetric volatility and extreme risk makes it particularly valuable for risk management during periods of market stress.
The analysis of seasonal volatility and tail risk provides actionable insights for investors, portfolio managers, and policymakers. By anticipating periodic increases in market risk and incorporating tail risk measures into risk management frameworks, stakeholders can better prepare for potential market downturns. The study's focus on Q4 2024 and Q1 2025 makes it particularly relevant for forecasting near-term market conditions.
Future research could explore the integration of additional external variables, such as macroeconomic indicators and geopolitical events, to further enhance the predictive power of volatility models. Additionally, the application of machine learning techniques could provide new insights into the complex dynamics of financial markets.

References

1. ***Engle, R. F. (1982)***. *Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50(4), 987-1008.*

2. ***Bollerslev, T. (1986)**. Generalized Autoregressive Conditional Heteroskedasticity. Journal of Econometrics, 31(3), 307-327.*
3. ***Box, G. E. P., & Jenkins, G. M. (1976)**. Time Series Analysis: Forecasting and Control. Holden-Day.*
4. ***Andersen, T. G., Bollerslev, T., & Diebold, F. X. (2003)**. Modeling and Forecasting Realized Volatility. Econometrica, 71(2), 579-625.*
5. ***Ghysels, E., Santa-Clara, P., & Valkanov, R. (2006)**. Predicting Volatility: Getting the Most Out of Return Data Sampled at Different Frequencies. Journal of Econometrics, 131(1-2), 59-95.*
6. ***McNeil, A. J., Frey, R., & Embrechts, P. (2015)**. Quantitative Risk Management: Concepts, Techniques, and Tools. Princeton University Press.*
7. ***Jorion, P. (2006)**. Value at Risk: The New Benchmark for Managing Financial Risk. McGraw-Hill.*
8. ***McNeil, A. J. (1997)**. Estimating the Tails of Loss Severity Distributions Using Extreme Value Theory. ASTIN Bulletin, 27(1), 117-137.*
9. ***Coles, S. (2001)**. An Introduction to Statistical Modeling of Extreme Values. Springer.*
10. ***Danielsson, J., & de Vries, C. G. (2000)**. Value-at-Risk and Extreme Returns. Annales d'Économie et de Statistique, 60, 239-270.*
11. ***Hansen, P. R., & Lunde, A. (2005)**. A Forecast Comparison of Volatility Models: Does Anything Beat a GARCH(1,1)? Journal of Applied Econometrics, 20(7), 873-889.*
12. ***McNeil, A. J., & Frey, R. (2000)**. Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: An Extreme Value Approach. Journal of Empirical Finance, 7(3-4), 271-300.*
13. ***Bali, T. G., & Weinbaum, D. (2007)**. A Comparative Study of Alternative Extreme Value Volatility Estimators. Journal of Financial Markets, 10(1), 1-33.*

These references provide a solid foundation for the methodologies and models used in this study, and they are essential for further reading and research in the field of financial volatility and risk management.